The Dynamics of Broadband Technology Diffusion in Western Europe *†

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Abstract
This study examines Western European broadband market equilibria within a simultaneous demand and supply model. Unlike alternative reduced-form approaches, our model explicitly incorporates key features of the demand for network services: a bell-shaped distribution of heterogeneous consumers; and the presence of network effects. Our empirical analysis of Western European countries reveals that although the demand for broadband services exhibit strong network effects; they are not ‘strong enough’ for critical mass to be achieved within these markets. Furthermore, the systems equilibrium dynamics show that initially high service prices lead suppliers to increase infrastructure investment, however these prices fall rapidly and substantially in the immediately following five-year period. Interestingly, these network effects stimulate expansion of the subscription base until consumers at the peak of the distribution enter the market. Additionally, convergence to the steady-state happens at around 8 years for service price and 15 years for subscription. Moreover, comparative-static analysis illustrates that steady-state equilibrium subscription is positively related to national population density, inter-platform concentration, and the share of unbundled local loop. By way of contrast, equilibrium price is inversely related to these same factors. Other supply and demand factors (viz., intra-platform concentration, income, and education) have minimal impact on the steady-state equilibrium.

JEL Classification: C53; L14; M37; L96

Keywords: critical mass; network effects; technology diffusion; broadband infrastructure; panel model

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1. Introduction

The introduction of broadband technology has enabled the Internet to encompass portable device, mobile, phone, and tablet computer networks (OECD, 2012). Naturally, the claimed benefits from this introduction of broadband technology are usually substantial; for example Czernich et al. (2011) report that a 1% point increase in national broadband penetration enhances annual per capita GDP growth by 0.09-0.15% points. Interestingly, most of these studies employ diffusion models to replicate the S-shaped paths observed for successfully adopted network commodities. A primary research task is the specification of location and speed parameters as functions of demographic, institutional, and other exogenous factors. Economic theory suggests that national income, population density, and inter-firm competition should foster broadband diffusion. However, the empirical impact of inter-platform competition and education on diffusion remains unclear (Gruber and Koutroumpis, 2013; Lee et al., 2011b; Dauvin and Grzybowski, 2014).

The insights obtained from these received empirical analyses are somewhat qualified by several commonly employed assumptions. In particular, the reduced-form approach imposes an S-shaped diffusion path. While an S-shaped diffusion path is proposed to result from the combination of a bell-shaped distribution of heterogeneous consumers and the presence of network effects, neither the shape of diffusion path nor the existence of attributes has been tested empirically for broadband markets. Accordingly, this study examines the dynamics of the broadband market equilibrium by simultaneously modelling market demand and supply. Uniquely, our simultaneous modelling approach does not impose an S-shaped diffusion path on the dynamics of broadband subscription. Rather, we specify structural broadband demand and supply functions, and derive both equilibrium price and subscription endogenously as depending on past subscriptions.

On the demand side, we follow Grajeck and Kretschmer (2012) in explicitly incorporating two key features of the demand for telecommunication services: a bell-shaped distribution of heterogeneous consumers and allowance for the presence of network effects. Also, our empirical model allows for cross-sectional and temporal variation in a bell-shape distribution of the consumers’ preferences by specifying them as a function of income and other demographic factors. On the supply side, we specify supply of broadband services as a function
of broadband price as well as institutional and demographic factors in a fashion similar to Röller and Waverman (2001) and Koutroumpis (2009).

Two main analyses are conducted on our simultaneous model of broadband demand and supply. First, we examine conditions for the existence of network critical mass, i.e., the subscription base beyond which the technology is self-perpetuating. Accordingly, we follow Grajeck and Kretschmer (2012) to implement a test on the (inverse) steady-state demand derived by equating current and past subscription rate in the structural demand function. Next, we endogenously derive equilibrium price and subscription as functions of past subscription by equating broadband demand and supply. Then, we examine how the dynamics of the market equilibrium, specifically, the steady-state equilibrium and speed of convergence to the steady state, differ across countries by demographic and institutional factors. From this analysis, we draw policy implications to most effectively foster broadband technology adoption.

Our simultaneous model is intended to extend received analysis on the diffusion of network technology in three important ways. First, our simultaneous-modelling approach does not impose an S-shape diffusion path on the dynamics of broadband subscription. Rather, the shape of the diffusion path as well as, the number, levels, and stability of steady-state equilibria are determined by the initial subscription, strength of the network effect, and exogenously-determined demographic and institutional factors. In particular, our model allows for multiple steady-state equilibria that can be either within or at the boundary of zero to full saturation range. This approach contrasts with the logistic diffusion model, which allows diffusion speed to differ by the demographic and institutional factors, yet imposes two possible convergence points to be at zero and full saturation only.\(^1\)

Second, previous analysis that employ reduced-form approaches have only focused on the dynamics of subscription, and not the price of broadband service. By contrast, in our case, the equilibrium subscription and service price are both endogenously determined. Our analysis hence addresses the equilibrium dynamics of both subscription rate and price and how these dynamics differ across countries by their demographic and institutional factors.

\(^1\) Although the maximum market size at full saturation can be made endogenous to the estimation model, previous studies commonly set it equal to population size or the number of households.
Finally, our simultaneous-model also makes several other important contributions to the empirical literature examining the existence of critical mass for telecommunication markets. For instance, in examining the existence of the critical mass for cellular phone markets in Europe, Grajeck and Kretschmer (2012) focused on the demand side of the market, and treated the price of cellular service as exogenous to their model. Furthermore, because the critical mass is equivalently defined as an unstable steady state, Grajeck and Kretschmer’s empirical analysis of demand dynamics is limited to the range of prices over which the steady states exist. By contrast, the equilibrium subscription rate and service price are endogenously determined in our simultaneous model. Our model hence allows a more complete analysis of the dynamics of the market equilibrium and how it responds to changes in the exogenous factors.

An empirical application of the model to a panel data from 14 west European countries reveals that the estimated demand function exhibits strong positive network effect, yet not to the level to assure the existence of critical mass.

Our empirical analysis of Western European countries reveals that although the demand for broadband services exhibit strong network effects; they are not ‘strong enough’ for critical mass to be achieved within these markets. Furthermore, the systems equilibrium dynamics show that initially high service prices lead suppliers to increase infrastructure investment, however these prices fall rapidly and substantially in the immediately following five-year period. Interestingly, these network effects stimulate expansion of the subscription base until consumers at the peak of the distribution enter the market. Additionally, convergence to the steady-state happens at around 8 years for service price and 15 years for subscription.

Moreover, comparative-static analysis illustrates that steady-state equilibrium subscription is positively related to national population density, inter-platform concentration, and the share of unbundled local loop. By way of contrast, equilibrium price is inversely related to these same factors. Other supply and demand factors (viz., intra-platform concentration, income, and education) have minimal impact on the steady-state equilibrium. The steady-state equilibrium exhibits substantial variations across 14 west European countries, due primarily to their difference in the values of demographic and institutional variables.

The rest of this paper is structured as follows. Section 2 provides a brief review of previous studies on the diffusion of broadband technology and identifies key limitations of the reduced-form approach commonly taken in the literature. Section 3 presents a simultaneous model of
broadband demand and supply. Section 4 describes data and reports the results from estimating the demand and supply model developed in Section 3. Section 5 analyses the estimated demand and supply functions and considers their implications for the presence of critical mass, the steady-state market equilibrium, the dynamics of the market equilibrium, and their cross-sectional variation and sensitivity to exogenously determined demographic and institutional factors. Section 6 concludes the paper with a synopsis of our analysis.

2. Selective Literature Review

Early studies of broadband diffusion necessarily analyse cross-section or short panel data (Lee et al., 2011a; Distaso et al., 2006; Bouckaert et al., 2010; Cava-Ferroruela and Alabau-Muñoz, 2006). Naturally, these data did not allow any consideration of process dynamics. However tentative the conclusions, the studies report that inter-platform competition (competition across platforms) fosters broadband subscription whereas intra-platform competition (competition across firms operating on the same platform) has no effect on subscription. The proposed effects of other control variables vary. For instance, Lee et al. (2011a) analyse OECD cross-national data and report income, education and ‘light form’ local loop unbundling (LLU) increase fixed-broadband diffusion. In contrast, Cava-Ferroruela and Alabau-Muñoz (2006) whilst reporting a positive income effect, find education has no influence.

An implicit assumption of static models is that broadband subscription responds only to variations in institutional, demographic, and other external factors. More recent analyses alleviate this limitation and explicitly analyse dynamics of the diffusion process. It is widely perceived that a successful network technology typically exhibits an s-shaped diffusion path, which is theoretically thought to be consistent with the case where heterogeneous consumers are distributed unevenly and network effects are ‘sufficiently strong.’ Operationally, logistic and Gompertz distributions are often assumed to replicate s-shaped diffusion processes with location and speed parameters specified as functions of demographic, institutional and other exogenous factors.

Studies employing dynamic approaches report mixed results as to effects of institutional and demographic factors on broadband diffusion. For example, Gruber and Koutroumpis (2013) using a logistic diffusion model report income and inter-firm competition foster diffusion, while inter-platform competition hinders diffusion. Also, applying logistic diffusion to a panel
data of OECD countries, Lee et al. (2011b) report inter-platform competition hinders fixed-broadband diffusion, while LLU policy, income, population density and education correlate with greater diffusions. Employing NUTS 1 European region data, Dauvin and Grzybowski (2014) report inter-platform competition, intra-platform competition, and income foster broadband diffusion, whereas education has a negative impact. Lin and Wu (2013) estimate Gompertz models on data from 34 OECD countries, and allow coefficients to vary by stage of diffusion. They report income, education, and Internet content drive early diffusion, while platform competition and price are important during middle and late stage diffusion.

While these empirical studies employing reduced-form approach have brought useful insights on the factors affecting the diffusion of broadband technology, they are subject to some limitations. First, these reduced-form diffusion models usually impose an s-shaped process, rather than allowing its key attributes, namely a bell-shaped distribution of consumers and network effect, to determine the diffusion process. Consequently, these models only allow two possible steady states, either zero or full saturation, with the converged state determined by the sign of the speed parameter. Furthermore, whilst the magnitude of market saturation can be estimated empirically, it is typically set to the population size or number of households. Consequently, potential market size is not allowed to vary by control variables.

Second, reduced-form models only describe the dynamics of broadband subscription; with broadband price playing no role in diffusions. Thus, these models neither draw implications on broadband price dynamics nor provide behavioral interpretations on model coefficients. Albeit, while some analyses include service price as exogenous arguments, the approach not only creates both endogeneity issues but also makes the model incomplete with the estimated coefficients representing only the net impacts, viz., total impact net of any indirect (price) impact.

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2 For EU member countries, a hierarchy of three NUTS levels is established by Eurostat; subdivisions do not necessarily correspond to administrative divisions within countries. A NUTS code begins with a two-letter country code, which is identical to the ISO 3166-1 alpha-2 code (except UK instead of GB for the United Kingdom and EL instead of GR for Greece). The country subdivision is referred to with one number. A second or third subdivision level is referred to with another number each. Each number starting with 1, as 0 is used for the upper level. Where the subdivision has more than nine entities, capital letters are used to continue numbering.

3 Gompertz diffusion models allow maximum market size, at saturation, to depend on demographic, institutional and external factors. These estimating equations form an autoregressive process of order one in the logistic transform of subscription, which is similar to the equilibrium subscription function derived from our structural model. However, the structural-form approach enable model coefficients to be given behavioral interpretations.
One way to address these limitations of reduced-form approach is to model demand and supply side in a structural form. An example of such structural-form modelling of demand for network service is provided by Grajeck and Kretschmer (2012). Their model of cellular phone demand explicitly incorporates the two key features of the s-shaped diffusion curve by assuming a bell-shaped distribution of consumer’s preference parameter and specifying utility from subscription to depend on the installed base. Within this framework, they determine conditions under which critical mass exists. Specifically, critical mass occurs on an increasing segment of the steady-state inverse demand curve, which requires a bell shaped distribution of consumers and ‘sufficiently strong’ network effects (when mobile technology is introduced).

The concept of critical mass is appealing but it reflects only one aspect of the diffusion process of network products. In particular, Grajeck and Kretschmer define critical mass as an unstable steady state, and analyse demand dynamics at (exogenous) prices for which steady states exist. However, while network products are often introduced at high prices that fall rapidly with diffusion, steady states do not exist until subscription reaches some critical point. In this situation, existence of critical mass is irrelevant for the successful adoption. Thus, to adequately characterise broadband diffusion requires both market demand and supply be included to allow equilibrium subscription and price to be endogenously determined.

3. Modeling Broadband Demand and Supply

This section develops a model of broadband demand and supply. On the demand side, we extend Grajeck and Kretschmer (2012)’s structural demand model by: (a) assuming a bell-shaped distribution of the preference parameter \(v\); and (b) introducing cross-sectional variation in the distribution of \(v\) through specifying it as a function of income and other demographic factors. On the supply side, we follow Röller and Waverman (2001) and Koutroumpis (2009), and specify broadband infrastructure and services as depending on demographic and institutional factors.

Our demand model starts with specifying consumers utility from broadband subscription as,

\[
u(v, x_{t-v})
\]

Critical mass is network size beyond which product diffusion is self-perpetuating, and also referred as the catastrophe point by Cabral (1990, 2006) and Shy (2001).
where \( v \) represents the consumer’s preference ordering. In particular, utility is a function of prior network size,

\[
(1) \quad u(v, X_{i,t-\delta}) = \exp \left( v + b_1 \lambda^{-1} \left( X_{i,t-\delta} \right) + b_2 \left( \lambda^{-1} \left( X_{i,t-\delta} \right) \right)^2 \right)
\]

where \( X_{i,t} = x_{i,t} / \text{Pop}_{i,t} \) is the subscription to population ratio, and \( \lambda^{-1}(z) = \ln \left( \frac{1}{z-1} \right) \) is a logit function.\(^5\)

Consumers subscribe only when the utility from subscription exceeds or equals service price. Thus, when consumers are ordered by preference parameter \( v \), the infra-marginal consumer is defined by,

\[
(2) \quad u\left( v^*, X_{i,t-\delta} \right) = \exp \left( v^* + b_1 \lambda^{-1} \left( X_{i,t-\delta} \right) + b_2 \left( \lambda^{-1} \left( X_{i,t-\delta} \right) \right)^2 \right) = p_{i,t}
\]

where \( p_i \) is price. Arranging (2) allows the infra-marginal consumer’s \( v \) to be expressed in terms of current price and prior period network size,

\[
(3) \quad v^* = \ln p_{i,t} - b_1 \lambda^{-1} \left( X_{i,t-\delta} \right) - b_2 \left( \lambda^{-1} \left( X_{i,t-\delta} \right) \right)^2.
\]

Unlike Grajeck and Kretschmer (2012), we assume \( v \) is distributed logistic with cumulative distribution function (CDF),

\[
(4) \quad F(v; m_{i,t}, s) = \frac{1}{1 + \exp \left( - \frac{v - m_{i,t}}{s} \right)}.
\]

where \( m_{i,t} \) and \( s \) are parameters determining the mean and variance of the distribution, respectively.

Since consumers whose preferences satisfy \( v > v^* \) subscribe, the share of consumers subscribing to the network is \( 1 - F(v^*) \). Thus,

\[
\]

\(^5\) Or the inverse of the logistic function \( \lambda(z) = \exp(z)(1 + \exp(z))^{-1} \).
(5) \[ 1 - F(v^*) = \exp \left( \frac{-v^* - m_{i,t}}{s} \right) \left( 1 + \exp \left( \frac{-v^* - m_{i,t}}{s} \right) \right)^{-1} = \lambda \left( \frac{-v^* - m_{i,t}}{s} \right) = X_{i,t} \]

Substituting (3) into (5) and arranging yields,

(6) \[ \lambda^{-1} \left( X_{i,t} \right) = \frac{m_{i,t} - v^*}{s} = \frac{1}{s} \left( m_{i,t} - \ln p_{i,t} + h_1 \lambda^{-1} \left( X_{i,t-\delta} \right) + h_2 \left( \lambda^{-1} \left( X_{i,t-\delta} \right) \right)^2 \right) \]

For our empirical analysis, we let \( m_{i,t} \) depend on selected demographic variables,

(7) \[ m_{i,t} = m_0 + m_1 \ln \frac{GDP_{i,t}}{Pop_{i,t}} + m_2 EDUP_{i,t} + m_3 GFC_i \]

where \( GDP/Pop \) is per capita gross domestic product, \( EDUP \) is combined secondary and tertiary enrolments to population ratio, and \( GFC \) is a dummy variable equal to unity during the GFC (2008: and 2008:4), and zero otherwise.

Substituting (7) into (6) yields our broadband demand estimating equation,

(8) \[ \lambda^{-1} \left( X_{i,t} \right) = \beta_{0,i} + \beta_1 \ln \frac{GDP_{i,t}}{Pop_{i,t}} + \beta_2 EDUP_{i,t} + \beta_3 GFC_i + \beta_4 \ln p_{i,t} + \beta_5 \lambda^{-1} \left( X_{i,t-\delta} \right) + \beta_6 \left( \lambda^{-1} \left( X_{i,t-\delta} \right) \right)^2 \]

where \( \beta_k = m_k s^{-1} \), for \( k = 0, i \) and \( k = 1, \ldots, 3 \), \( \beta_4 = -s^{-1} \), \( \beta_5 = h_1 s^{-1} \), and \( \beta_6 = h_2 s^{-1} \).

In (8), \( \beta_4 < 0 \) is expected as \( s \) (standard deviation of \( v \)) should be positive. The other three variables in the right-hand side of (8) directly impact on the mean of the preference parameter. Since higher \( v \) values indicate greater willingness to subscribe, we expect \( \beta_1 > 0 \) (normal good) and \( \beta_2 > 0 \) (subscription increases with education), and \( \beta_3 < 0 \).

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It is worth noting that while the logistic distribution of \( v \) guarantees the model is consistent with the assumed bell-shape distribution, it is also econometrically desirable as the dependent variable is the logistic transform of the subscription rate \( X \), whose value is not bounded from below or above. Thus, estimation of (8) is not subject to econometric issues associated with truncated dependent variable, unlike the model using the subscription rate \( X \) as a dependent variable, whose value is bounded between 0 and 1.
On the supply side, we follow Röller and Waverman (2001) and Koutroumpis (2009), and specify broadband infrastructure and service supply as,

\[ \ln BBinv_{it} = a_{i0} + a_{1} InterP_{it} + a_{2} IntraP_{it} + a_{3} Reg_{it} + a_{4} \ln PopDen_{it} + a_{5} GFC_{it} + a_{6} \ln P_{it} \]  

(9)

\[ \lambda^{-1}(X_{it}) - \lambda^{-1}(X_{it-d}) = c_0 + c_1 \ln BBinv_{it} + c_2 \lambda^{-1}(X_{it-d}) \]  

(10)

where \( BBinv \) is capital invested in broadband infrastructure, \( InterP \) and \( IntraP \) are measures of inter-platform and intra-platform concentration, respectively. \( Reg \) is measures market regulation, \( PopDen \) is population per square kilometres of land area, and \( GFC \) is the Global Financial Crisis dummy variable. Equation (9) is more flexible than Koutroumpis (2009) due to the inclusion of the three additional arguments \( Reg, PopDen, \) and \( GFC. \) Finally, broadband infrastructure depreciation (if \( c_2 < 0 \)) depends on past subscription.

The market concentration variables (\( InterP, IntraP \)) are measured by the Herfindahl-Hirschman index (HHI). Higher values in the interval \([0, 1]\) represent more concentrated broadband supply. Thus, \( a_1 < 0 \) and \( a_2 < 0 \) are expected when competition across firms or platforms fosters supply. \( Reg \) is measured by the LLU to copper lines ratio, and so \( a_3 > 0 \) is expected.

For the other variables, we expect that higher population density enables broadband provision at lower cost, hence attracting broadband infrastructure investment. Further, we expect firms would invest more at high service prices and less during the GFC. Thus, \( a_4 > 0, a_5 < 0, \) and \( a_6 > 0 \) are expected.

Combining (9) and (10) yields,

\[ \hat{\lambda}^{-1}(X_{it}) = a_{i0} + a_{1} InterP_{it} + a_{2} IntraP_{it} + a_{3} Reg_{it} + a_{4} \ln PopDen_{it} + a_{5} GFC_{it} + a_{6} \ln P_{it} + a_{7} \lambda^{-1}(X_{it-d}) \]  

(11)

where \( a_{i0} = c_0 + c_{a_{i0}}, \) \( a_k = c_1 a_k \) for \( k = 1, \ldots, 6, \) and \( a_7 = c_1 (1 + c_2). \) As for demand, prior penetration is an argument in (11). Our empirical analysis determines the optimal lag order by minimizing the sum of squared residuals.
4. Data

Data for empirical estimation are obtained from various sources. The *Telecoms Market Matrix (Western Europe)* dataset produced by Analysys Mason Limited, provides data on selected mobile, broadband and narrowband telecommunications. These data cover 14 OECD Member Countries including: Austria, Belgium, Denmark, Finland, France, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom for 2003:4 through 2013:4. The other databases are the World Bank *World Development Indicators* (WDI) and the OECD statistics database, from which demographic information is obtained.

Data from the *Telecoms Market Matrix* are used to construct panels for broadband subscription rate (percentage of the population subscribed to broadband service by platform). Service price is constructed for each country as quantity-weighted revenue per subscriber by broadband service provided,

\[
P_{i,t} = \sum_{m=1}^{M_{i,t}} \frac{rev_{m,i,t}}{sub_{m,i,t}}
\]

where \( rev_{m,i,t} \) is total platform \( m \) service provider revenue, \( sub_{m,i,t} \) is platform \( m \) subscription, and \( M_{i,t} \) are the available platforms, with subscripts \( i \) and \( t \) indicating the country and period of the observation, respectively.

The measures of intra- and inter-platform concentration are constructed as:

\[
InterP_{i,t} = \sum_{m=1}^{M_{i,t}} \left( \frac{X_{m,i,t}}{X_{i,t}} \right)^2
\]

\[
IntraP_{i,t} = \sum_{n=1}^{N_{i,t}} \left( \frac{X_{n,i,t}}{X_{i,t}} \right)^2
\]

where \( X_{m,i,t} \) and \( X_{n,i,t} \) are, respectively, subscription to platform \( m \) and subscribers receiving broadband from firm \( n \). \( M_{i,t} \) and \( N_{i,t} \) are the number of platforms and firms in

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7 Germany and Greek are included in the dataset yet removed from our empirical study due to the limited availability of their demographic data.

8 Analysys Mason list subscription in five categories: DSL, cable modem, FTTH/B, Broadband FWA and other fixed platforms.
country $i$, respectively. $X_{i,t} = \sum_{m=1}^{M_{i,t}} X_{m,i,t} = \sum_{n=1}^{N_{i,t}} X_{n,i,t}$ represents total broadband subscription.

Finally, we follow Koutroumpis (2009) and measure the degree of ULL regulation as the share of incumbent broadband available to operators, to total cooper lines.

From the WDI and OECD databases we source: per capita GDP (measured in euro), combined secondary and tertiary enrolment to total population, and population density (population per square kilometre). Data are reported annually, and conversion to quarterly series is made via cubic spline. In total, 504 observations from 14 countries over 36 quarters are obtained.  

Table 1 provides summary statistics. The sample average subscription and service price are 0.56 percent and 94.23 euro, respectively. Both series vary substantially, with subscription ranging from 0.03 to 0.91 percent, and price from 51.75 to 159.96 euro. The last two columns of the table indicates that variation is mostly temporal for subscription, and cross country for price.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Std. Dev. across countries</th>
<th>Std. Dev. across time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subscription percent</td>
<td>0.56</td>
<td>0.2</td>
<td>0.03</td>
<td>0.91</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>Price euro</td>
<td>94.23</td>
<td>22.8</td>
<td>51.8</td>
<td>160.0</td>
<td>19.13</td>
<td>5.69</td>
</tr>
<tr>
<td>Inter-platform comp. index</td>
<td>0.60</td>
<td>0.2</td>
<td>0.35</td>
<td>0.96</td>
<td>0.15</td>
<td>0.02</td>
</tr>
<tr>
<td>Intra-platform comp. index</td>
<td>0.35</td>
<td>0.1</td>
<td>0.19</td>
<td>0.69</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Regulation ULL/wirelines</td>
<td>0.10</td>
<td>0.1</td>
<td>0.00</td>
<td>0.48</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>GDP euro</td>
<td>50464.35</td>
<td>11317</td>
<td>24984</td>
<td>90195</td>
<td>10008</td>
<td>5567</td>
</tr>
<tr>
<td>Education enrolment/pop</td>
<td>0.12</td>
<td>0.01</td>
<td>0.10</td>
<td>0.15</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Population density pop/1,000 km²</td>
<td>154.85</td>
<td>131.5</td>
<td>12.6</td>
<td>498.6</td>
<td>136.24</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Data on GDP are not available for 2013 while those on education variable are not available for Germany for entire period.
Table 2 provides sample statistics of the two key variables by country and year. It reveals that over 36 quarters, average subscription triples from 0.25 (2004) to 0.73 (2012) while service price falls by 12%. The minimum and maximum national subscription values also range widely over time: for France, Ireland, Norway, Switzerland and the UK subscriptions increase by 60 percentage points through the period. We also observe wide variation in initial subscriptions, which range from 3% (Ireland) to over 30% (Belgium, Denmark, Netherlands and Switzerland).

Service price on the other hand exhibits substantial cross-country variation both in its mean and standard deviation. Large cross-sectional variation in price is apparent in the annual price spreads. It is also seen in large standard deviations across countries calculated each year, which range from 19.00 (2008) to 27.46 (2012). Thus, while price gradually trends down through the observation period, the national temporal variations are less even, with a price standard deviation ranging from 4.07 for Switzerland to 25.48 for UK.
Table 1 also reports standard deviations for the inter-platform concentration, intra-platform concentration and regulation variables from 0.09 to 0.15. However, the inter-platform concentration index exhibits substantially more cross-country (0.15) than temporal (0.02) variation. This contrasts with the temporal and cross-section variations of the intra-platform concentration (0.06, 0.07) and regulation (0.07, 0.05) variables. Finally, per capital GDP and population density variations are mostly cross-country.

5. Estimation

Since the demand and supply equations in (8) and (11) both allow cross-sectional variation in the constant term, they are estimated via the fixed effects model. Furthermore, the price on the right-hand side of the two equations is endogenous. Thus, the models are estimated by generalized instrumental variable (GIV) methods using exogenous variables from the system but excluded selectively from each equation as instruments. For the lag length of the network effect in each of the demand and supply equations (δ and d, respectively), we estimate each model for the lag length varying from 1 to 8 quarters. The minimum sum of squared residuals adjusted by sample size finds, a one-period lag optimal for both equations.

Table 3 summarizes the demand and supply equation fixed effects GIV estimation results. The estimated demand and supply functions fit the observed data very closely, with $R^2 = 0.998$ for both equations. In contrast, the estimated price equation in the first-stage of IV estimation yields an $R^2 = 0.438$, indicating relative difficulty of explaining cross-sectional and temporal variation in the observed broadband price. The reported F and Wald statistics indicate that arguments are jointly significant for each equation. The calculated F statistic also supports these fixed effects model specifications.

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10 These instruments include InterP, IntraP, Reg, and PopDen for the estimation of demand equation, and GDP/Pop, EDU, and squares of lagged penetration for the estimation of supply equation.

11 Our criterion differs from Grajeck and Kretschmer (2012) who determine the optimal lag of the network effect requiring the regression residual exhibit no significant serial correlation. They are concerned that the lagged subscription coefficient reflect serial correlation in subscription rather than network effects. By minimizing serial correlation in residual term, one might risk capturing serial correlation via the lagged subscription terms. We therefore allow serial correlation in residual when determining the optimal lag. The optimal lag length of one quarter, however, corresponds to the one considered by Grajeck and Kretschmer’s (2012) empirical analysis of European cellular phone markets, and Koutroumpis’ (2009) analysis of the economic impact of broadband.

12 $R^2$’s are obtained from the first-difference estimator of the fixed effects model, and represent the share of variation accounted for by all arguments after removing cross-sectional variation after first-differencing the reduced-form price equation.
The first-stage price regression estimates indicate that price increases with income (GDP). This finding is expected as more affluent consumers demand more services, and the increased cost of meeting incremental demand results in higher market-clearing prices. Also, the negative coefficient for the population density is consistent with the notion that broadband service is provided at lower cost in more densely concentrated populations. Interestingly, the coefficients are negative for the concentration variables, suggesting that concentrated markets deliver broadband service at lower prices. The effect is more pronounced for inter-platform concentration, with the estimated coefficient being seven-fold of that for intra-platform concentration.13

For the estimated demand equation, the estimated price (negative) and per capita GDP (positive) coefficients are signed according to priori expectations, yet the latter is insignificant. For the network effect, the estimated coefficients are positive for both linear and quadratic term,
with corresponding structural estimates: \( b_1 = -\beta_b / \beta_a = -0.938 / (-0.077) = 12.149 \) and \( b_2 = -\beta_b / \beta_a = -0.011 / (-0.077) = 0.142. \)

The first- and second-derivatives of consumer’s utility,

\[
\frac{\partial u(v, X_{i,t-\delta})}{\partial X_{i,t-\delta}} = \frac{u(v, X_{i,t-\delta})}{X_{i,t-\delta}(1-X_{i,t-\delta})} \left( b_1 + 2b_2 \lambda^{-1}(X_{i,t-\delta}) \right)
\]

\[
\frac{\partial^2 u(v, X_{i,t-\delta})}{\partial X_{i,t-\delta}^2} = \frac{u(v, X_{i,t-\delta})}{X_{i,t-\delta}(1-X_{i,t-\delta})^2} \left[ 2b_2 \left( b_1 + 2b_2 \lambda^{-1}(X_{i,t-\delta}) \right) \right]
\]

which are both positive for \( b_1 > 1 \) and \( b_2 > 0 \) for \( 0 < X < 1 \), indicating that the network effect increases with subscription at an increasing rate.\(^{15}\) For the other two exogenous variables, the education coefficient is not significant, while the GFC (negative) coefficient is signed according to expectations.

For the estimated supply equation, the coefficient for log price is positive, while it is near unity at 0.943 for the lagged subscription rate, which translates to 5.7% depreciation rate in logistic transform of the subscription rate. The estimated coefficient for LLU regulation is positive (0.10) as expected. However, contrary to our expectation, positive coefficients are estimated for the two concentration variables, it is large positive and significant for inter-platform concentration and insignificant for intra-platform concentration.

### 6. Equilibrium Paths

Next, we consider equilibrium subscription and price dynamics. We start by analysing the demand-side market dynamics, particularly focusing on the existence of critical mass. We then

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\(^{14}\) The standard error of these structural-form parameters are 5.12 and 0.05, making \( b_1 \) and \( b_2 \) statistically significant at 5% and 1% size, respectively.

\(^{15}\) The utility function is specified as a quadratic function of the logit transform of \( X \), while Grajeck and Kretschmer (2012) specify the utility as a quadratic function of \( X \) (without logit transform). The difference in the reported shape of the network effects between the two studies, however, is not due to the difference in the functional form of the utility function. The two functional forms both allow the network effect to be either convex or concave, with the possibility of the critical point to be at the boundary.
incorporate supply side into our analysis and examine the dynamics of the market equilibrium and how it is affected by shocks from exogenous variables.

Figure 1. Illustrative broadband demand phase diagram

Typically, broadband demand dynamics can be portrayed on the $X_{t-\delta}, X_t$ plane. In the diagram, intersections of the subscription curve with the 45 degree line define steady states at which subscription stays unchanged through time.\(^{16}\) Figure 1 illustrates the case where network effect is substantially strong and the consumer’s preference parameter $v$ is distributed bell-shaped. In the figure, the subscription rate increases (decreases) where the curve lies above (below) the 45 degree line. Consequently, of the three steady states, the one in the middle (corresponding to $X_1, X_{t-\delta}$) is unstable whereas the other two are stable. Grajeck and Kretschmer (2012) equivalently defines this unstable steady state as the critical mass. Importantly, the curve’s shape as well as the number of steady states and their stability are determined by the magnitude of network effect, distribution of $v$, and values of the exogenous variables and price.

\(^{16}\) Steady-state demand is alternatively termed ‘fulfilled expectations demand’ by Economides and Himmelberg (1995).
Figure 2 plots the estimated demand functions evaluated at each of the three levels (sample mean, minimum and maximum) of service price (while setting the values of per capita GDP, education, and GFC dummy at their respective sample means). All three curves exhibit positive and s-shaped relationship between current and past subscription rate. They intersect the 45 degree line from above and below and these intersections define the stable and unstable steady state, respectively.

When evaluated at the sample mean price, the stable and unstable steady states are found at $\bar{X}_{t-1} = 0.878$ and $0.975$, respectively. When the curve is evaluated the maximum price, the two steady states decrease and increase to $0.6618$ and $0.9932$, respectively. As price falls the steady states converge to the single steady stat. This happens when $P = 82.57$ (at the sample

---

17 The upward sloping curve in Figure 2 is consistent Grajeck and Kretschmer’s (2012) lemma 1. It states the slope of $X_t$ in the $X_{t-d}$ space is determined by network externalities $du'(v, X_{t-d})/dX_{t-d}$, which are always positive for the estimated demand equation.

18 The negative estimated price coefficient causes the vertical position of the curve to be inversely related to price. Naturally, the first and second steady state relates inversely and directly to price, respectively.
mean of the exogenous variables). The curve does not intersect the 45 degree line at lower prices, so subscription increases until market saturation \((X_s = 1)\) is reached.

**Steady-State Inverse Demand**

The relationship between the price and subscription rate at the steady state can be compactly expressed by setting \(X_t = X_{t-1} = X\) on equation (8),

\[
\ln p_{t,t} = \frac{1}{\beta_4} \left( \mu_{t,t}(Z, \psi) + (1 - \beta_5) \lambda^{-1}(X_t) - \beta_6 \left( \lambda^{-1}(X_t) \right)^2 \right) .
\]

which defines the steady-state inverse demand function. The curve shows that demand depends on the strength of network effects, distribution of \(v\), and price.\(^{19}\)

Grajeck and Kretschmer demonstrate that to attain critical mass requires equation (13) be upward-sloping, at least partially, in the interval \(0 < X < 1\). Clearly, the shape of the inverse demand curve depends on the external market factor (which determine \(\mu_{t,t}\)) and \(\beta\) values, given the distribution \(v\) and utility function specification.\(^{20}\)

Figure 3 graphs steady-state demand evaluated at the demographic variable sample averages. With demand convex and downward sloping (for most of \(0 < X < 1\)), critical mass does not exist in European broadband markets. The critical point of this inverse demand function defines the lowest price for which the steady state exists.\(^{21}\) It is obtained by solving,

\[
\frac{\partial \ln p_{t,t}}{\partial X_t} = \frac{1}{\beta_4} \left( 1 - \beta_5 - 2 \beta_6 \lambda^{-1}(X_t) \right) \frac{1}{X_t(1-X_t)} = 0 .
\]

\(^{19}\) Grajeck and Kretschmer (2012) emphasize that the dependence of subscription on price is what differentiates the structural-form model from the reduced-form model of Bass (1969), who specified diffusion as depending on the difference between current subscription, and market capacity.

\(^{20}\) Specifically, \(\mu_{t,t}\) affects the vertical positioning of the inverse demand curve, whereas the shape and horizontal location depends on \(\beta_4, \beta_5, \) and \(\beta_6\).

\(^{21}\) More formally, one can check the sign of the second derivative of the inverse demand function with respect to \(X\). See appendix B.
The internal solution is \( X_i = \lambda \left( (1 - \beta_x) / 2 \beta_x \right) \), which evaluated at \( \hat{\beta}_x = 0.938 \) and \( \hat{\beta}_b = 0.011 \), is \( \bar{X} = 0.944 \). The associated price, 82.57, is the minimum price for the existence of the steady state.\(^{22}\)

![Figure 3. Steady-state demand](image)

Table 4 reports the responsiveness of the steady-state inverse demand equation to changes in the price, GDP, education and GFC variables. First, the derivative of steady-state subscription with respect to price is \( \partial X / \partial \ln P = -0.336 \) at sample average subscription. That is, the steady-state demand is inelastic, and falls by only 0.0034 percentage points in response to a 1% price increase.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Effect</th>
<th>t-ratio*</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>-0.336</td>
<td>-2.614</td>
</tr>
<tr>
<td>per capita GDP</td>
<td>0.028</td>
<td>0.244</td>
</tr>
<tr>
<td>education</td>
<td>0.707</td>
<td>0.315</td>
</tr>
<tr>
<td>GFC</td>
<td>-0.069</td>
<td>-2.121</td>
</tr>
</tbody>
</table>

\(^{**}, ***\) denote coefficients are significant at 5% and 1%, respectively. t ratio calculated with delta method standard errors.

---

\(^{22}\) The price below which the \( X_i, X_{i-d} \) curve does not intersect the 45 degree in Figure 2.
For changes in the exogenous demographic variables, per capita GDP and the education variable have no measured impact, whereas steady-state demand was lower by 7 percentage points during the GFC period.

Market Equilibrium

Our analysis of critical mass and steady-state demand has treated price as exogenous. To fully consider market equilibrium dynamics requires that supply be both incorporated into the model. Solving the demand and supply equations in (8) and (11) define the equilibrium price and subscription as,

\begin{align*}
P_{it}^* &= \exp \left\{ \eta \left( \theta_{it} - \mu_{it} + (\alpha_7 - \beta_5) \lambda^{-1} (X_{it,\theta}) - \beta_6 (\lambda^{-1} (X_{it,\theta}))^2 \right) \right\} \\
X_{it}^* &= \lambda \left\{ \eta \left( \beta_4 \theta_{it} - \alpha_6 \mu_{it} + (\alpha_7 - \alpha_6 \beta_5) \lambda^{-1} (X_{it,\theta}) - \alpha_6 \beta_6 (\lambda^{-1} (X_{it,\theta}))^2 \right) \right\}
\end{align*}

where \( \eta = (\beta_4 - \alpha_6)^{-1} \). Importantly, solutions (15) and (16) depend on the demographic variables (as they determine \( \theta_{it} \) and \( \mu_{it} \)) and past subscription values.

Figure 4. Equilibrium price and subscription path
Figure 4 plots the equilibrium price and subscription time paths with GDPC, PopDen, EDU, GFC, IntraP, InterP, and Reg measured at the sample mean. Equilibrium subscription exhibits standard s-shaped dynamics. By way of contrast, equilibrium price after declining steeply (from around 300 euro), levels off at around 100 euro. Importantly, during the transition to generally lower price, equilibrium subscription grows to around 35%. Interestingly, after quarter 16, continued network growth (to about 85%) cannot be attributed to the continued decline in prices. Plausibly, these subsequent subscription increases are attributable to strong network effects on the demand side.

Figure 5. Equilibrium market clearing price and subscription path

The systems’ equilibrium subscription dynamics can also be considered by plotting current against past subscription. In particular, Figure 5 compares estimated equilibrium subscription values obtained from equation (16) against those from the demand curve (8). Importantly, while the estimated subscriptions are evaluated at the exogenous variable means, they face different price values. That is, equilibrium subscription responds to the time-varying equilibrium price determined by (15), whereas price is exogenous (evaluated at the sample mean) for the subscription demand equation. While these plots are similar, the market equilibrium path is

23 The figures are drawn over 100 quarters, starting with 1% subscription rate in the initial period, which is well below the minimum value of 3.3% observed in the data.
slightly more concave, and is attributable to the market equilibrium price being substantially higher than the sample average when subscription is initially low.

The steady-state equilibria are obtained by setting \( X_{i,t} = X_{i,t-\delta} = X_i \) and solving for \( \lambda^{-1}(X_i^*) \)

\[
\lambda^{-1}(X_i^*) = -\frac{\alpha_6 \beta_5 - \alpha_7 \beta_4 - \alpha_6 + \beta_4}{2\alpha_6 \beta_6} \pm \sqrt{\left(\alpha_6 \beta_5 - \alpha_7 \beta_4 - \alpha_6 + \beta_4\right)^2 - 4\alpha_6 \beta_6 \left(\alpha_6 \mu_{i,t} - \beta_4 \theta_{i,t}\right)}.
\]

The solutions define a stable steady-state equilibrium and an unstable steady state. Substitution yields the steady-state equilibrium price,

\[
P^* = \exp\left(\theta_{i,t} - \mu_{i,t} + (\alpha_7 - \beta_5) \lambda^{-1}(X_i^*) - \beta_6 \left(\lambda^{-1}(X_i^*)\right)^2\right).
\]

Evaluating at the sample mean of the exogenous variables yields stable 0.8504 (97.78 euro) and unstable 0.9989 (775.74 euro) steady-state equilibrium subscriptions (prices).

**Comparative Statics**

The steady-state market equilibria defined in (17) and (18) depend on the values of the demographic and other external factors. The comparative-statics of the steady-state equilibrium are, for the demand factors,

\[
\frac{\partial X_i^*}{\partial Z} = -\frac{\alpha_6 \beta_5 X_i^*(1-X_i^*)}{2\alpha_6 \beta_6 \lambda^{-1}(X_i^*) + \left(\alpha_6 \beta_5 - \alpha_7 \beta_4 - \alpha_6 + \beta_4\right)}
\]

and for the supply factors,

\[
\frac{\partial X_i^*}{\partial Z} = \frac{\beta_4 \alpha_7 X_i^*(1-X_i^*)}{2\alpha_6 \beta_6 \lambda^{-1}(X_i^*) + \left(\alpha_6 \beta_5 - \alpha_7 \beta_4 - \alpha_6 + \beta_4\right)}.
\]

with the effect on equilibrium price given by,

\[
24 \text{ See Appendix B for derivations.}
\]
\[
\frac{\partial \ln P^*_i}{\partial Z_{j,t}} = -\frac{1}{\alpha_6 - \beta_4} \frac{\partial (\theta_{i,t} - \mu_{i,t})}{\partial Z_{j,t}} - \frac{2\beta_5 \lambda^{-1}(X^*_i)}{(\alpha_6 - \beta_4) X^*_i (1 - X^*_i)} \frac{\partial X^*_i}{\partial Z_{j,t}}
\]

where \( \frac{\partial (\theta_{i,t} - \mu_{i,t})}{\partial Z_{j,t}} = -\beta_j \) for demand and \( \frac{\partial (\theta_{i,t} - \mu_{i,t})}{\partial Z_{j,t}} = \alpha_j \) for supply factors.

Table 5 summarizes the comparative statics of the market equilibrium with respect to each of the seven external factors, evaluated at the exogenous variable means. On the demand side, per capita GDP and education positively affect the steady-state equilibrium subscription. However, the GDP impact is small with a 1% increase in per capita GDP raising equilibrium subscription by 0.018 percentage points. The effect on steady-state equilibrium price is also small with a 1% increase in per capita GDP only raising equilibrium price by 0.03%. By contrast, a percentage point increase in the secondary and tertiary enrolment share increases steady-state equilibrium subscription (price) by 0.45 (0.75) percentage points.\(^{25}\)

Table 5. The Comparative-statics of Market Equilibrium

<table>
<thead>
<tr>
<th>Unit change</th>
<th>Standard deviation change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subscription</td>
<td>Price</td>
</tr>
<tr>
<td>Demand</td>
<td></td>
</tr>
<tr>
<td>GDP per capita</td>
<td>0.018</td>
</tr>
<tr>
<td>Enrolment (% population)</td>
<td>0.453</td>
</tr>
<tr>
<td>GFC</td>
<td>-0.044</td>
</tr>
<tr>
<td>Supply</td>
<td></td>
</tr>
<tr>
<td>Inter-platform competition index</td>
<td>0.387</td>
</tr>
<tr>
<td>Intra-platform competition index</td>
<td>0.021</td>
</tr>
<tr>
<td>Regulation (% ULL in copper lines)</td>
<td>0.148</td>
</tr>
<tr>
<td>Population density</td>
<td>0.583</td>
</tr>
<tr>
<td>GFC</td>
<td>-0.028</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>GFC - Demand + Supply</td>
<td>-0.072</td>
</tr>
</tbody>
</table>

\(^{25}\) The standard deviation of the education variable is 0.01. Thus, a percentage point change is equivalent to a standard deviation change. Accordingly, the corresponding 0.45 percentage point change in subscription translates to a 0.02 standard deviation change. Similarly, 0.75% increase in price is roughly equal to a 0.03 standard deviation increase.
On the supply side, a one unit increase in inter-platform concentration (reduced competition from perfectly competitive market to a pure monopoly) leads to a 38.8 percentage point increase in subscription. However, a unit increase in the competition index results in a 1.15% price reduction. Intra-platform concentration has similar, but smaller, impacts on equilibrium price and subscription. A unit increase in the intra-platform concentration index increasing subscription by 2 percentage points, while price falls by 0.06%.

For regulation, a percentage point increase in the ULL share raises equilibrium subscription by 0.147 percentage points, and lowers price by 0.44%. Also, increased (by 1%) population density raises equilibrium subscription by 0.583 percentage points, and lowers equilibrium price by 1.74%.

Finally, the GFC period corresponded to lower equilibrium subscription via both the demand and supply channels. The total impact is unambiguously negative with the subscription decreased by 7.2 percentage points during the GFC. The situation is more complex for the steady-state equilibrium price; the GFC lowered the price 7.3% through the demand channel while it increased the price by 8.4% via the supply channel. The net effect was a 1% increase in equilibrium price.

**Steady-State Market Equilibrium**

The comparative-static analysis reported above implies that countries in varied demographic and institutional settings exhibit different steady-state market equilibria. We find the extent of national inter-platform concentration and population density have substantially greater impacts on the steady-state market equilibrium than other exogenous factors.

In Table 6, the steady-state market equilibriums, evaluated at the national sample averages of the external factors, exhibit wide variation across the sampled nations. The steady-state subscription rate ranges from 63.7% to 98.6%, while service price ranges from 68.88 euro to 202.33 euro. Interestingly, higher steady-state subscription rates generally coincide with relatively high service prices.

Importantly, the reported cross-sectional variations in the steady-state equilibrium are complex. In particular, the revealed steady-state equilibrium patterns are comprised of the above comparative-static components, but also depend on the values of the country-specific fixed effects in the estimated demand and supply equations. For instance, the value of the fixed effect
in the estimated demand function relates positively to steady-state equilibrium price and subscription rate. Inspection of the second last (Demand) column of Table 6, reveals that the fixed effect in the estimated demand function is large positive for Denmark, France, Netherlands, Norway, and Switzerland, which results in high steady-state subscription (from 93.8% to 98.6%) and service price (ranging from 139.77 euro to 202.33 euro) for these countries. In contrast, the estimated demand function fixed effects are large negative for Austria, Italy, and Portugal, resulting in low subscription (from 62.7% to 65.4%) and service price (from 68.88 euro to 76.21 euro) at the steady-state equilibrium.

Table 6. Steady-state Market Equilibrium, at Sample Mean

<table>
<thead>
<tr>
<th>Country</th>
<th>Subs</th>
<th>Price</th>
<th>GDPPC</th>
<th>Edu.</th>
<th>Pop.</th>
<th>Inter</th>
<th>Intra</th>
<th>Reg.</th>
<th>Demand</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.64</td>
<td>73.8</td>
<td>52036</td>
<td>0.13</td>
<td>100.8</td>
<td>0.52</td>
<td>0.35</td>
<td>0.09</td>
<td>-0.07</td>
<td>-0.01</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.84</td>
<td>126.2</td>
<td>48689</td>
<td>0.11</td>
<td>355.8</td>
<td>0.51</td>
<td>0.37</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.49</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.95</td>
<td>174.2</td>
<td>51378</td>
<td>0.13</td>
<td>129.7</td>
<td>0.45</td>
<td>0.43</td>
<td>0.07</td>
<td>0.06</td>
<td>-0.07</td>
</tr>
<tr>
<td>Finland</td>
<td>0.80</td>
<td>89.0</td>
<td>47599</td>
<td>0.14</td>
<td>17.5</td>
<td>0.61</td>
<td>0.24</td>
<td>0.10</td>
<td>-0.02</td>
<td>0.64</td>
</tr>
<tr>
<td>France</td>
<td>0.97</td>
<td>139.8</td>
<td>45992</td>
<td>0.13</td>
<td>117.7</td>
<td>0.89</td>
<td>0.32</td>
<td>0.25</td>
<td>0.04</td>
<td>-0.10</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.83</td>
<td>87.2</td>
<td>55722</td>
<td>0.12</td>
<td>64.3</td>
<td>0.60</td>
<td>0.37</td>
<td>0.02</td>
<td>-0.01</td>
<td>0.18</td>
</tr>
<tr>
<td>Italy</td>
<td>0.63</td>
<td>68.9</td>
<td>42729</td>
<td>0.11</td>
<td>199.9</td>
<td>0.94</td>
<td>0.42</td>
<td>0.16</td>
<td>-0.07</td>
<td>-0.38</td>
</tr>
<tr>
<td>Netherlands*</td>
<td>0.99</td>
<td>202.3</td>
<td>53727</td>
<td>0.13</td>
<td>489.2</td>
<td>0.50</td>
<td>0.30</td>
<td>0.15</td>
<td>0.06</td>
<td>-0.52</td>
</tr>
<tr>
<td>Norway</td>
<td>0.94</td>
<td>178.3</td>
<td>75768</td>
<td>0.13</td>
<td>13.2</td>
<td>0.54</td>
<td>0.35</td>
<td>0.13</td>
<td>0.05</td>
<td>0.74</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.65</td>
<td>76.2</td>
<td>32409</td>
<td>0.10</td>
<td>115.1</td>
<td>0.48</td>
<td>0.45</td>
<td>0.06</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>Spain</td>
<td>0.72</td>
<td>93.8</td>
<td>41441</td>
<td>0.11</td>
<td>91.2</td>
<td>0.66</td>
<td>0.37</td>
<td>0.09</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.76</td>
<td>97.6</td>
<td>50990</td>
<td>0.13</td>
<td>22.6</td>
<td>0.43</td>
<td>0.28</td>
<td>0.09</td>
<td>-0.02</td>
<td>0.57</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.97</td>
<td>141.9</td>
<td>60946</td>
<td>0.11</td>
<td>194.7</td>
<td>0.57</td>
<td>0.36</td>
<td>0.03</td>
<td>0.04</td>
<td>-0.18</td>
</tr>
<tr>
<td>UK</td>
<td>0.93</td>
<td>82.9</td>
<td>47074</td>
<td>0.12</td>
<td>256.2</td>
<td>0.64</td>
<td>0.27</td>
<td>0.18</td>
<td>0.01</td>
<td>-0.31</td>
</tr>
<tr>
<td>Overall</td>
<td>0.85</td>
<td>97.8</td>
<td>50464</td>
<td>0.12</td>
<td>154.9</td>
<td>0.60</td>
<td>0.35</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Steady-state equilibrium not defined for Netherlands at the sample means, and the estimated demand and supply functions. The reported subscription rates and price levels are maximums for stable steady state.

Conversely, the fixed effect in the estimated supply function is negatively related to the steady-state equilibrium price and positively to subscription. From the last (Supply) column of Table 6, the fixed effect in the estimated supply equation are particularly large for Finland, Norway, and Sweden, which result in relatively high subscription rates (80.1%, 93.8%, and 76.4%), even though extremely low national population densities might suggest very low subscriptions. Finland and Sweden achieve high steady-state subscriptions at moderate service prices (88.97 euro and 97.62 euro). By contrast, steady-state equilibrium price is very high for Norway (178.25 euro), due to high steady-state national subscription and a large positive demand fixed effect (which is positively related to the steady-state equilibrium price and subscription).
The comparative-static analysis of the previous section suggests that countries with relatively high population densities (Belgium, Italy, Netherlands, Switzerland, and the UK) should expect relatively high subscriptions and relatively low service prices at the steady-state equilibrium. However, the UK and Switzerland are the only countries that reveal this combination, with the UK achieving the above sample-average subscription (92.6%) at below average price (89.19 euros), whilst Switzerland achieves very high subscription (97.4%) at not excessively high price (141.85 euros). For the remaining countries, the link between the high population density and the steady-state equilibrium is masked by large negative fixed effects in the estimated supply equation, which result in relatively high steady-state service prices (126.17 euro, 68.88 euro, and 202.33 euro for Belgium, Italy, and Netherlands, respectively) and the corresponding national subscriptions (84.0%, 62.7%, and 98.6%, respectively).

The above comparative-static analysis also suggested that countries with relatively high inter-platform concentration (France and Italy) would have higher subscriptions at moderate service prices. However, this positive impact on steady-state subscription is offset by large negative fixed effects in the estimated demand and supply functions for Italy, resulting in low steady-state equilibrium subscription (62.7%) at low service price (68.8 euro). For France, the positive impact of inter-platform competition on the steady-state subscription rate is amplified while the negative impact on the steady-state equilibrium price is offset by a large fixed effect in the estimated demand function (which positively impacts on both the steady-state equilibrium subscription and service price), resulting in high national subscription (97.0%) at moderate price level (139.77 euro).

Equilibrium Path

Equation 6 and Figure 7 illustrate, respectively, the dynamics of equilibrium subscription and service price as predicted by the estimated demand and supply functions for sampled nations during the period. For each subscription and price path, two predictions are plotted by country-time pair. The first prediction is based on the subscription observed in the previous period and the exogenous variables in the current period. This one-period ahead prediction is calculated using the rules,

\[
\hat{X}_{i,t}^* = \mathbb{E}[X_{i,t} | X_{i,t-1}, Z_{i,t}, \hat{\alpha}, \hat{\beta}]
\]

\[
= \lambda \left( \hat{\eta} \left( \hat{\beta}_d \mu_{i,t} - \hat{\alpha}_d \mu_{i,t} + \left( \hat{\alpha}_c \hat{\beta}_c - \hat{\alpha}_c \hat{\beta}_c \right) \lambda^{-1} \left( X_{i,t-1} \right) - \hat{\alpha}_c \hat{\beta}_c \left( \lambda^{-1} \left( X_{i,t-1} \right) \right)^2 \right) \right)
\]
\[
\hat{P}_{i,t}^* = E[P_{i,t} | X_{i,t-1}, Z_{i,t}, \hat{a}, \hat{\beta}]
= \exp\left\{ \hat{\eta} \left( \theta_{i,t} - \mu_{i,t} + (\hat{\alpha}_7 - \hat{\beta}_5) \lambda^{-1} \left( X_{i,t-5} \right) - \hat{\beta}_6 \left( \lambda^{-1} \left( X_{i,t-6} \right) \right)^2 \right) \right\}
\]

where \( \hat{\eta} = (\hat{\beta}_4 - \hat{\alpha}_5)^{-1} \), \( \hat{a} \) and \( \hat{\beta} \) are, respectively, the coefficient vectors in the estimated supply and demand function, and \( Z_{i,t} \) is the vector of exogenous variables that determine the values of \( \theta_{i,t} \) and \( \mu_{i,t} \).

The second prediction is based on the subscription at the initial period and the current observed values of the exogenous factors,

\[
\hat{X}_{i,t}^* = E[X_{i,t}^* | X_{i,t}, Z_{i,t}, \hat{a}, \hat{\beta}]
= \lambda \left( \hat{\eta} \left( \hat{\beta}_4 \theta_{i,t} - \hat{\alpha}_6 \mu_{i,t} + (\hat{\alpha}_7 - \hat{\beta}_5) \lambda^{-1} \left( \hat{X}_{i,t-1}^* \right) - \hat{\alpha}_6 \hat{\beta}_6 \left( \lambda^{-1} \left( \hat{X}_{i,t-1}^* \right) \right)^2 \right) \right)
\]

\[
P_{i,t}^* = E[P_{i,t}^* | X_{i,t}, Z_{i,t}, \hat{a}, \hat{\beta}]
= \exp\left\{ \hat{\eta} \left( \theta_{i,t} - \mu_{i,t} + (\hat{\alpha}_7 - \hat{\beta}_5) \lambda^{-1} \left( \hat{X}_{i,t-1}^* \right) - \hat{\beta}_6 \left( \lambda^{-1} \left( \hat{X}_{i,t-1}^* \right) \right)^2 \right) \right\}
\]

where \( \hat{X}_{i,t}^* = X_{i,t} \) for \( t = 2 \) and the prediction for previous period equilibrium subscription rate \( \hat{X}_{i,t-1}^* \) is successively updated for \( t > 2 \).
Figure 6. Observed and predicted national equilibrium broadband subscription
Figure 6 indicates that both predictions of national subscription closely follow observed subscription. Any deviations represent a mixture of model errors in predicting equilibrium subscription and deviations of actual subscription from the equilibrium rate. Interesting, deviations are smaller for the one-period ahead prediction than for that based on the initial period subscription. This latter observation implies that past penetration on the right-hand side of the estimated demand equation captures not only the network effect but also serial correlation of the subscription, as discussed by Grajeck and Kretschmer (2012) and Hartmann et al. (2008). Predictions based on initial subscription exhibits greater deviations from observed values, yet the deviations are of reasonable size, considering that predictions are made over as many as eighty quarters ahead, and that they partially reflect deviations of actual subscription from the true equilibrium rate.

The model’s predictions of equilibrium price in Figure 7 replicate long-term trend of the observed price reasonably well. Noticeably, countries at the initial stage of diffusion in the beginning of the study period (e.g., Ireland, Italy, and the UK) commonly exhibit rapid price reduction initially. Other nations, with more advanced initial diffusions at the beginning of the sample period (e.g., Belgium, Denmark, the Netherlands, and Switzerland) commonly indicate gradual fall in the service price, followed by an increase.

The model’s predictions, however, do not trace short-term fluctuations of the observed price series very well. Unlike national subscription, the one period-ahead price prediction does not improve accuracy over the price prediction based on the initial period subscription rate. This result is not surprising given relatively low R-square obtained in the first-stage GIV estimation. As for the subscription, the deviation between the predicted and observed price represents a mixture of the prediction error and the deviation of the observed price from the true equilibrium price. In addition to these components, a considerable amount of the deviation can be attributed to measurement error from approximating the service price by the ratio of total revenue aggregated over companies operating on different platforms to total subscribers.
Figure 7. Observed and predicted national equilibrium broadband service price

Austria  Ireland  Spain  
Belgium  Italy  Sweden  
Denmark  Netherlands  Switzerland  
Finland  Norway  UK  
France  Portugal


- $P_{\text{obs}}$
- $P_{\text{pred}(0)}$
- $P_{\text{pred}(1)}$
6. Conclusion

In this study, we examined the dynamics of the broadband technology diffusion in several Western European markets. Our approach employed a simultaneous broadband demand and supply model that explicitly allowed for consumers with heterogeneous preferences and the presence of network effects. To incorporate these innovations, we relied on aspects of the seminal structural-form model of Grajeck and Kretschmer (2012). In particular our demand model: (1) assumed a bell-shaped distribution of consumers’ preference parameter (which is consistent with standard theoretical models of technology diffusion); (2) explicitly incorporated socioeconomic variables to account for cross-sectional variation in the distribution of the consumer preferences). Additionally, our specification also incorporates supply-side features of the broadband market in a manner similar to that proposed by Röller and Waverman (2001), and Koutroumpis (2009). Also, by explicitly specifying the network effect, our model allows us to examine the critical mass exists empirically. In particular, our simultaneous modelling approach allows us to consider the dynamics of the market equilibrium (for both subscription and price) and any sensitivity the external market factors.

Our empirical analysis revealed that Western European broadband service demand is both own-price and income inelastic. Furthermore, and not surprisingly, broadband service demand also exhibits strongly positive network effects. Interestingly, the effect does not attenuate with increased subscription. However, this network effect is not sufficiently strong to assure the existence of network critical mass.

Our analysis of the equilibrium dynamics indicates that broadband technology is introduced at initially very high service prices. High service prices attracts further infrastructure investment, which drives equilibrium price downward in the five years after technology. Whilst price falls slow around year five, subscription continues to expand due strong network effects, which cause gradual increases in service price. Ultimately, network subscription growth slows after consumers at the peak of the preference parameter distribution enter the market. Also, convergence to the steady-state equilibrium occurs at around 8 years for service price and 15 years for the subscriber base.

Our comparative-static analysis illustrated that steady-state equilibrium price and subscription depend on both demographic and institutional factors. In particular, equilibrium subscription is related positively to population density and inter-platform concentration, while the
equilibrium service price is relates inversely to these factors. The share of unbundled local loop also affects equilibrium subscription (positively) and service price (negatively). Other supply (intra-platform concentration) and demand (income and education) factors only have a minimal impact on the steady-state market equilibrium.

Finally, we observe that the nature of steady-state equilibrium achieved exhibits substantial variations across Western European nations, which is attributed to national demographic and institutional factors. Thus, cross-sectional variation in the estimated fixed effects contributes to variation in the predicted steady-state equilibriums. To draw richer policy implications and maximize social returns from broadband investment and subscription we require the model to explicitly incorporate additional demographic and institutional variables that impact on demand and supply. The extension of the presented study in this direction will be left for the future analysis.

References


Cabral, L. (1990), ‘On the Adoption of Innovations with ‘Network’ Externalities,’ *Mathematical Social Sciences* 19, 299-308


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Appendix

A1: Steady-state demand critical point

Since the critical point is at the boundary of \( X \), the inverse demand function is monotonic over the relevant range of \( X \). The second derivative,

\[
\frac{\partial^2 \ln p_i}{\partial X_i^2} = -\frac{1}{\beta_4} \left( (1 - \beta_5 - 2\beta_6 \lambda^{-1}(X_i))(1 - 2X_i) + 2\beta_6 \right) \left( \frac{1}{X_i(1 - X_i)} \right)^2
\]

(evaluated at \( \bar{X}_i = \lambda \left( (1 - \beta_5) / 2\beta_6 \right) \)) is,

\[
\frac{\partial^2 \ln p_i}{\partial X_i^2} = -\frac{2\beta_6}{\beta_4} \left( \frac{1}{X_i(1 - X_i)} \right)^2.
\]

Thus, solution (12) is the global maxima (minima) when \( \beta_4 \beta_6 > 0 \) (\( \beta_4 \beta_6 < 0 \)). The coefficients \( \hat{\beta}_4 = -0.077 \) and \( \hat{\beta}_6 = 0.011 \), \( \hat{\beta}_4 \beta_6 < 0 \), identify equilibrium subscription at the globally minimum price.

A2: Comparative-static Analysis of Market Equilibrium

Comparative-static analysis of the steady-state with respect to the exogenous variables obtained from,

\[
\frac{\partial \bar{X}_i}{\partial Z} = -\frac{\partial C(\bar{X}_i, Z; \Phi)}{\partial Z} \left( \frac{\partial C(\bar{X}_i, Z; \Phi)}{\partial \bar{X}_i} \right)^{-1}
\]

where \( C(X, Z; \Phi) \) equals

\[
C(X, Z; \Phi) = \alpha_6 \beta_6 \left( \lambda^{-1}(X_i) \right)^2 + \left( \alpha_6 \beta_5 - \alpha_7 \beta_4 - \alpha_6 + \beta_4 \right) \lambda^{-1}(X_i) + \alpha_6 \mu_{i,i} - \beta_4 \theta_{i,i} = 0
\]

and

\[
\frac{\partial C(\bar{X}_i, Z; \Phi)}{\partial \bar{X}_i} = \frac{2\alpha_6 \beta_5 \lambda^{-1}(X_i) + \left( \alpha_6 \beta_5 - \alpha_7 \beta_4 - \alpha_6 + \beta_4 \right)}{X_i(1 - X_i)}
\]
The first term on the right-hand side of (A1) for the demand arguments is,

$$\frac{\partial C(X, Z; \Phi)}{\partial Z_{j,i,t}^d} = \alpha_6 \frac{\partial \mu_{i,t}}{\partial Z_{j,i,t}^d} = \alpha_6 \beta_j$$

and for the supply arguments,

$$\frac{\partial C(X, Z; \Phi)}{\partial Z_{j,i,t}^s} = -\beta_4 \frac{\partial \theta_{j,i,t}}{\partial Z_{j,i,t}^s} = -\beta_4 \alpha_j.$$  

Using these expressions, the comparative-static equilibrium price effects are,

$$(A2) \quad \frac{\partial \ln \bar{p}_i}{\partial Z_{j,i,t}} = \frac{1}{\alpha_6 - \beta_4} \frac{\partial}{\partial Z_{j,i,t}} \left( \theta_{i,t} - \mu_{i,t} \right) - \frac{\left( \alpha_6 - \beta_4 \right)}{\left( \alpha_6 - \beta_4 \right) \bar{X}_i \left( 1 - \bar{X}_i \right)} \frac{\partial \bar{X}_i}{\partial Z_{j,i,t}}$$

where $\frac{\partial (\theta_{i,t} - \mu_{i,t})}{\partial Z_{j,i,t}^d} = -\beta_j$ for the demand factor $\frac{\partial (\theta_{i,t} - \mu_{i,t})}{\partial Z_{j,i,t}^s} = \alpha_j$ for the supply factor.