Aggregating Labour Supply and Feedback Effects in Microsimulation

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Abstract
This paper explores an extension of behavioural microsimulation modelling so that third round effects of a policy change can be simulated. The first round effects relate to fixed hours of work, while second round effects allow for changes in desired hours of work at unchanged wages. Third round effects allow for endogenous changes to the distribution of wage rates resulting from the labour supply responses to tax changes. This is achieved using the concept of an aggregate ‘supply response schedule’, which identifies the extent to which average labour supply responds to a proportional change in wage rates. The third round effect is obtained after re-running a microsimulation model with a suitable modification to individuals’ wage rates. The method is illustrated using the MITTS behavioural microsimulation model.

1. Introduction
Behavioural microsimulation models for the analysis of tax policy changes are designed to examine the effects on government expenditures, taxation and labour supply of a tax policy change, allowing for the considerable complexity of individuals’ budget constraints.1 These models are partial equilibrium supply side models in which the wage rate distribution is fixed. However, substantial changes in labour supply as a result of a non-marginal tax policy change may give rise to changes in the wage rate distribution, depending on the demand for labour. It is therefore desirable to be able to accommodate such wage changes, at least to some extent.2

The ‘first round’ effects of a tax policy change refer to tax changes which hold labour supplies fixed. The ‘second round’ effects allow for labour supply responses, with wage rates held constant. This paper considers the question of how to take behavioural microsimulation analysis one stage further, by modelling possible aggregate effects on wages of tax policy

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1 For a survey of methods of dealing with labour supply responses in microsimulation models, see Creedy and Duncan (2002).
2 Bergmann (1990) discussed microsimulation models in which aggregate unemployment is generated, resulting from job search behaviour in the presence of unemployment insurance. However, all considerations relating to wage levels (on either supply or demand sides of the market) were ignored.

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Given a method of producing changes to the wage rate distribution, such changes can be fed back into the microsimulation model in order to obtain adjusted labour supply responses and government expenditure estimates: this gives the ‘third round’ effects. The proposed method involves a multi-stage procedure in which the simulated labour supply effects of a policy change are aggregated and combined with extraneous information about the demand side of the labour market. The method allows for different degrees of aggregation to be used. An alternative process of obtaining third round effects may involve the use of an independent computable general equilibrium multi-industry model. However, compatible models are seldom available and the method suggested here requires only extraneous information about labour demand elasticities.

A preliminary problem involves labour supply aggregation, for combining with an aggregate demand schedule. A straightforward correspondence between the output from a microsimulation model and a conventionally defined aggregate supply curve is not possible. The concept of a supply response schedule is introduced in section 2. The proposed procedure for simulating feedback effects is presented in section 3. An illustrative example is provided in section 4 using the Melbourne Institute Tax and Transfer Simulator (MITTS) for Australia.

2. Aggregating Labour Supply

The standard individual labour supply function cannot be used in the present context, as this is discontinuous at the point of entry into the labour market, and net-wage elasticities bear only a loose resemblance to gross-wage elasticities. Labour supply arises from maximisation of a direct utility function \( U(c_i, h_i; X_i) \), where \( h_i \) and \( c_i \) represent hours worked and consumption (or net income, where the price index is normalised to unity) for individual \( i \) with characteristics \( X_i \), subject to a budget constraint. The individual’s labour supply function, for variations in the gross wage, \( w \), takes the general form

\[
h'_i = h(w; X, T),
\]

where \( T \) refers to the complete tax and transfer system. This function is made up of segments based on interior solutions and those arising from kinks in the budget constraint where the effective marginal tax rate changes.

Individual labour supply functions display considerable variation in the wage elasticity of hours supplied.

Furthermore, aggregate supply functions discussed in the literature cannot be used in the present context. When estimating labour supply functions

3. The analysis remains static in the sense that cross-sectional information only is used and individuals are not treated as forming expectations about future wages and prices, or optimising within a life-cycle framework.

4. For discussion of individual labour supply functions, see Creedy (2001).

5. Some analyses of aggregate labour supply in macroeconomic models are based on the intertemporal substitution model of Lucas and Rapping (1969); see, also Altonji (1982, 1986) and Kennan (1988), who examines identification issues. The model is based on the argument that individuals form expectations about future real wages, in relation to current wages, based on movements in commodity prices. If real wages are considered to be high relative to expected future wages, current labour supply increases as a result of intertemporal substitution. The present approach excludes such dynamic considerations.
using time series data on variables such as average hours and real wage rates, it is difficult to deal with the types of response discussed above and of individual heterogeneity. The models typically involve no explicit treatment of aggregation issues.

Consider again the individual labour supply function (1). At the observed wage of \( w_i \) for person \( i \), the predicted labour supply when faced with tax system \( T = T_0 \) is:

\[
h'_i(w) = h(w_i; X_i, T = T_0)
\]

One approach to aggregation might average labour supply predictions over a range of wage rates, with each individual being presented with the same wage rate. Using sample weights \( g_i \) (representing the number of individuals of type \( i \) in the population), this results in an aggregate supply of the form:

\[
H'_s(w) = (\sum g_i)^{-1} \sum g_i h(w; X_i, T)
\]

at some common wage \( w \). However, the interpretation of a function representing the average of a set of individual labour supplies on the presumption that all are given the same wage is unclear. In practice, it would involve predictions of individual labour supplies at wage rates far from their observed wage.

The Supply Response Schedule

As stressed above, any method of dealing with ‘third round’ effects must handle the considerable heterogeneity which is the foundation of microsimulation modelling. As labour demand information is likely to be available, at best, for specified population groups (defined by, say, occupation or education), the approach adopted here is to adjust the wage rate distribution of the group. The approach involves the concept of a ‘supply response schedule’: this traces the variation in labour supply per capita as the wage rate distribution of the group undergoes proportional shifts, that is, the relative dispersion of wage rates within the group remains constant.

Given a particular tax and transfer system, the expected labour supply of each individual in a microsimulation model can be computed as the individual’s wage rate varies—using an appropriate behavioural modelling component. Starting from the individual’s observed wage rate, expected labour supply can be obtained for proportional reductions and increases in each individual’s wage (for a given tax structure). This can be carried out for all individuals in a specified population group, enabling the expected labour supply per capita in the group as a whole to be related to equal proportional changes in the wage rates all individuals. Thus the supply

\(^6\) Strong conditions are needed for aggregate labour supply and commodity demands, as functions of some index of prices and wages, to be consistent with individual optimisation. The use of a representative agent framework was examined by Deaton and Muellbauer (1980) and, in the context of labour supply by Muellbauer (1981), who abstracted from corner solutions.

\(^7\) Discussion of the extensive or intensive margin usually involves the use, in the econometric modelling, of either changes in average hours worked or changes over time in the number of individuals; see, for example, Alogoskoufis (1987).
response schedule indicates how average labour supply responds to shifts in the distribution of wage rates. This supply response schedule is therefore constructed using an extensive numerical simulation exercise, involving simulated labour supply responses to wage changes, conditional on a given tax and transfer structure. The individual supply responses are based on optimising behaviour, allowing for the full complexity of individuals’ characteristics and budget constraints resulting from the tax and transfer system.

The supply response schedule can be defined more formally as follows. A behavioural microsimulation model is used to obtain individuals’ hours responses to a proportionate change in observed wage rates. That is, the full wage distribution is perturbed, and the average labour supply response to that perturbation is obtained. The perturbation is based on an index \( i_w \in (-1, \infty) \) which replaces each individual’s wage \( w \) with an amount \( w(1+i_w) \). The response schedule is constructed by averaging individuals’ labour supply responses at a given index \( i_w \), giving a function, in per capita terms, of the form:\(^8\)

\[
H_s(i_w) = (\sum g_i^{-1} \sum h(w_i (1+i_w); X, T))
\]

For \( i_w = 0 \) this reduces to \( H_s(0) = (\sum g_i^{-1} \sum h(w_i; X, T)) \), which should be close to the observed aggregate = \( \overline{H} = (\sum g_i^{-1} \sum h_i) \).

The advantage of this type of supply response schedule is that each point on the schedule is consistent with a distribution of wages, together with the underlying tax and transfer scheme and population characteristics. Movement along the supply response schedule arises from a shift in the entire wage distribution. A schedule of this type is shown in Figure 1.

Figure 1  A Supply Response Schedule

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\(^8\) The average process may use sample weights provided by the cross-sectional survey on which the microsimulation model is built.
The supply response schedule thus traces the average labour supply resulting from equal proportionate changes in wage rates, conditional on the tax and transfer system and the nature of the population. It can be generated directly from a behavioural microsimulation model and provides the information needed to produce feedback effects of tax policy changes.

**Shifts in the Supply Response Schedule**

The approach can be used to simulate shifts in the supply response schedule resulting from a change in tax or welfare policy. Because the supply response schedule is built from the predictions of a behavioural microsimulation model, the effects of an enormous range of policy reforms could be simulated. Suppose there are two tax systems, \( T_0 \) and \( T_1 \), where \( T_0 \) represents some benchmark tax system corresponding to a given year’s microdata, and \( T_1 \) represents the tax system following a hypothetical policy reform. Let the predicted labour supplies of the \( i \)th individual at wage rate \( w(1+i) \) under systems \( T_0 \) and \( T_1 \) be denoted respectively by \( h_0^*(i,w(1+i);X,T = T_0) \) and \( h_1^*(i,w(1+i);X,T = T_1) \). By aggregating the difference between these predictions at any value of \( i \) in the manner of 4), a simulation of the shift in overall labour supply when moving from tax system \( T_0 \) to \( T_1 \) is obtained as

\[
\Delta H_s(i,w|T_0,T_1) = (\Sigma g) \sum_{i=1}^{\infty} [h_0^*(i,w) - h_1^*(i,w)].
\] 

The extent to which the wage rate distribution shifts in response to a change in the tax system is obtained by combining the supply response schedule with a demand for labour (per capita) schedule. This is described in the following section.

### 3. A Multi-stage Procedure

This section describes a multi-stage procedure used to produce third round effects of a policy change to the tax and transfer system, allowing for the effects on the distribution of wages.

**Modelling Feedback Effects**

The first round effects of a tax policy change are based on the assumption that there are no labour supply responses. The second round effects allow for labour supply responses but keep the wage rate distribution fixed. This is illustrated in Figure 2. It is equivalent to a vertical movement from an initial supply response schedule (under \( T_0 \)) to the revised schedule (for \( T_1 \)); that is, \( i_w = 0 \).

The previous section has described how a shift in the supply response schedule, as a result of a tax change, can be generated. This shift can be used, in combination with information about a corresponding aggregate demand response schedule for labour, to generate a suitable value of \( i_w \). This value of \( i_w \) is given by the intersection between the new supply response schedule and the demand response schedule. It can then be fed into a revised behavioural simulation to generate a new set of labour supply responses and government expenditures for the same tax policy change, using a shift in the wage rate distribution. These new responses give the third round simulation.9

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9 In principle, further iterations could be performed until convergence is reached.
Figure 2  A Shift in The Supply Response Schedule

\[ H_s'(i_w | T=T_0) \]

\[ H_s'(i_w | T=T_1) \]

\[ H_s'(0|T_0,T_1) \]

\[ i_w = 0 \]

Measuring Wage Effects in Microsimulation

The second round effect of a tax or welfare policy reform is equivalent to the interaction of the supply response schedule with an infinitely elastic labour demand schedule. Thus a change in labour supply is absorbed with no change in the distribution of wage rates. In the absence of further information, it is useful to consider the other extreme assumption of a completely inelastic demand schedule. That is, if aggregate supply increases, wages must fall proportionately in order to keep aggregate hours fixed at the initial level. This is illustrated in Figure 3. Suppose that the tax system shifts from \( T_0 \) to \( T_1 \) following a policy reform in a way which causes the supply response schedule to shift downwards. That is,

\[ H_s'(i_w | T=T_1) < H_s'(i_w | T=T_0) \text{ for all } i_w. \]  

(6)

With demand fixed at \( LD = \bar{LD} \), the supply shift increases the distribution of wages by \( i_{max} \). This factor may be used to adjust each individual’s wage before being fed back into the behavioural simulation model to produce the third round simulation.

With a downward sloping aggregate demand schedule of the sort shown in Figure 4, the same inward shift in aggregate supply produces movement both in the wage distribution and equilibrium supply. Under these circumstances, the third round shift in aggregate supply corresponds to the difference

\[ \Delta H_s = H_s'(i_w | T=T_1) - H_s'(0|T=T_0), \]  

(7)
where \( i_w^* \) is the difference in the intersections of the supply response schedules under \( T_0 \) and \( T_1 \) and the aggregate demand schedule.

The approach could also be applied to separate groups population, distinguished for example by education or occupation. This would produce a set of adjustment terms, enabling a change in the dispersion of wage rates to be modelled. A given policy change may produce wage increases for some groups while wages may fall for others, depending on the labour supply and demand effects.10

Figure 4  A Shift in The Supply Response Schedule: Flexible Demand

10 The wage adjustment model sketched here is only one among a range of possible models, including search-theoretic models, insider-outsider/efficiency wage/wage curve models which factor unemployment into the market structure. The method proposed in this paper could be extended to alternative models of labour market adjustment.
4. Some Illustrative Examples

This section illustrates the use of aggregate supply response schedules in behavioural microsimulation. The aim is mainly to demonstrate the potential of the approach, as in practice more disaggregated demand side information would be needed. In view of the difficulty of obtaining disaggregated data on demand functions, the results should be regarded as merely showing possible orders of magnitude and the way in which third round effects can be obtained. The examples are obtained using the Melbourne Institute Tax and Transfer Simulator (MITTS). A simple policy reform is examined in which the Australian income tax system of January 2000 is modified: the initial income tax rate is increased from 20 percent to 25 percent. The behavioural responses in MITTS are obtained using a discrete choice frame work involving a range of discrete hours levels. Quadratic utility functions were estimated for a range of demographic groups, and calibration is used to place each individual at the observed hours in the pre-reform situation, that is, the optimal hours equal the actual hours worked. For further details of the MITTS model, see Creedy et al. (2002).

Simulated Aggregate Supply Response Schedules

The aggregate supply response schedule for the full sample of working-age households was constructed using the formulation in (4). Labour supplies both under the baseline January 2000 system and the reform system were produced. The simulated average labour supply under the January 2000 system at the observed distribution of wages is broadly equivalent to the observed overall average labour supply, of around 19 hours per week. In Figure 5, this corresponds to the point on the first aggregate supply response schedule at which $i_w = 0$. Varying $i_w$ under the January 2000 system produces the aggregate supply response schedule.

Figure 5 shows that this empirical schedule is positive and monotonic over the range $i_w = [-0.5, 0.7]$. The empirical wage elasticity at the distribution of wage rates observed in the data ($i_w = 0$) was found to be around 0.62 for the full population, rising slightly as the distribution of wages increases beyond the observed level, before eventually falling for larger perturbations. At lower wage rates (corresponding to negative $i_w$), the aggregate wage elasticity falls, and is around 0.35 when all wages are halved. This decline in the aggregate wage elasticity might be caused in part by an increased reliance on transfer payments, particularly among low wage workers.

The formulation in (5) is used to simulate a shift in the aggregate supply response schedule following the increase of 5 percentage points in the basic income tax rate. In Figure 5, the post-reform schedule (the dashed line) is everywhere below the first schedule. At equivalent wage rates for all individuals in the sample, aggregate labour supply reduces following in the increase in taxes. At the observed distribution of wages, the average reduction in labour supply is around 0.36 hours (equivalent to around 2 percent).
Figure 5  Simulated Supply Response Schedules

Dissagregated Supply Response Schedules

It was suggested above that the approach can be applied to particular demographic groups, by averaging individual responses over target groups within the sample, rather than over the sample as a whole. Section (a) of Figure 6 shows several supply response schedules.

The first row in this series of simulations shows the comparative aggregate supply schedules of high- and low-education individuals, displaying systematically higher labour supply on average among the higher educated group. The corresponding elasticities are higher for the low-education group. This could be a manifestation of Heckman’s (1993) observation that higher labour supply responses are found at the extensive (participation) margin.

Section (c) of Figure 6 compares the supply response schedules of sole parents with those of the full sample. Average labour supply is systematically lower, a function in part of lower participation rates among sole parents. The shift in the supply response schedule for sole parents is proportionately larger, reflecting a higher sensitivity among this group to changes in the net wage. This is borne out by the wage elasticities, which are higher for sole parent households as wage rates fall. Similar patterns emerge when comparing the supply response schedules of women and men, as shown in section (b) of Figure 6.

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15 The high-education group corresponds to those with more than the basic (compulsory) level of education. This comparison was designed to proxy the relative supply response schedules of high and low skill labour in the absence of observations on skill levels for non-working individuals in the sample.
Figure 6(a) Disaggregated Supply Schedules

Figure 6(b) Disaggregated Supply Schedules
Simulating Wage Responses to a Policy Change

The previous section described a method by which to simulate the proportional effects on the wage distribution following a shift in the aggregate supply schedule. Under assumptions regarding the equivalent aggregate labour demand schedule, the intersections of simulated demand and supply might be used to model the extent to which wages might adjust following a policy reform. With inelastic labour demand, the extent of the proportionate shift in wage rates could be simulated by measuring the horizontal distance $i_{\max}^*$ between the two aggregate supply schedules at the pre-reform level of aggregate supply; see Figure 3. With a flexible demand response schedule, the full effect of the policy reform comprises shifts both in the wage distribution and in the equilibrium level of supply and demand; see Figure 4. Both cases are examined here.

For the simulations described earlier, the third-round effects of the 5 percentage point increase in income tax can be measured under alternative hypothetical specifications of aggregate labour demand. Consider two alternatives: the first assumes that demand is completely inelastic, so that $L_d(i_w) = L_0$ in the earlier formulation. The second assumes that $L_d(i_w) = L_0(1+i_w)\varepsilon_d$ with elasticity $\varepsilon_d = 0.5$, a central figure in the range of labour demand elasticities typically reported in the empirical literature; see Hamermesh (1993).

Figure 7 shows the relevant detail (focussing on the locality of the intersection) of the simulated aggregate supply response schedule, which applies to all individuals combined, and an aggregate demand schedule with elasticity $\varepsilon_d = 0.5$. For an inelastic labour demand schedule, the wage
effect $i_{max}$ is measured to be around 3.7 percent (the horizontal distance between the aggregate supply response and supply shift schedules, measured from the point of intersection of the pre-reform labour supply schedule at $i_w = 0$). The second assumption is that labour demand is responsive to changes in the wage distribution, with an elasticity of 0.5. From Figure 7, the effect of the tax policy change, on wages and labour supply/demand under the assumption of a clearing labour market, can be measured. The third round response combines a simulated increase of 2.07 percent ($i_w = 0.0207$) in the wage distribution with a reduction of 1.0 percent in aggregate labour supply.

Figure 7 Demand and Supply Response Schedules: Detail

Table 1 First, Second and Third Round Effects of Tax Reform

<table>
<thead>
<tr>
<th>Percentage change in:</th>
<th>First round</th>
<th>Second round</th>
<th>$\varepsilon_d=0$</th>
<th>$\varepsilon_d=0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax revenue</td>
<td>+9.1%</td>
<td>+7.9%</td>
<td>+13.4%</td>
<td>+11.7%</td>
</tr>
<tr>
<td>Allowance costs</td>
<td>-</td>
<td>+2.8%</td>
<td>-0.5%</td>
<td>+0.1%</td>
</tr>
<tr>
<td>Aggregate labour supply</td>
<td>-</td>
<td>-1.9%</td>
<td>-0.1%</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Wage rates</td>
<td>-</td>
<td>-</td>
<td>+3.7%</td>
<td>+2.1%</td>
</tr>
</tbody>
</table>

To summarise the successive effects of a change in the tax system, Table 1 shows the first, second and third round effects of the hypothetical tax change. The percentage change in income tax revenues, aggregate labour supply and wages at the three stages of simulation are reported. For the third round effects, results are reported for inelastic labour demand (providing an upper limit to the potential wage response) and for an aggregate demand elasticity of 0.5 (providing a more central simulation of the potential wage response).
The first column of Table 1 shows the first round effects of the income tax increase, with income tax revenues increasing by around 9 percent. The second column includes labour supply responses in the simulation. As reported in earlier sections, labour supply is predicted to reduce on average by 1.9 percent in response to this tax reform. These behavioural responses have further effects on government revenues and costs; the reduction in labour supply results in a lower increase in income tax revenues than in the first-round simulation (from 9.1 percent to 7.9 percent). There is an increase in the cost of allowance payments, of around 2.8 percent, as earnings fall.

The final column of Table 1 shows how the earlier responses might change following potential adjustments in the distribution of wage rates. For a perfectly inelastic labour demand schedule, the shift in aggregate supply results in a proportionate increase of around 3.7 percent in the distribution of wage rates. Feeding these wage increases back into an adjusted behavioural simulation, income tax revenues rise by 13.4 percent compared with the status quo. The cost of allowance transfers actually falls by 0.5 percent, since earnings increase with the adjustment in wages. As expected, aggregate labour supply remains effectively unchanged relative to the status quo, as is implied by the assumption of a fixed level of labour demand. With an aggregate labour demand elasticity of 0.5, the adjustment is less extreme. Wage rates are modelled to increase by 2.1 percent, and aggregate labour supply falls by around 1 percent overall. These effects combine to generate an increase of 11.7 percent in the revenue from income tax, and an increase of 0.1 percent in the cost of allowances. In this illustrative example, the same percentage wage change is applied to all individuals. However, the same basic method can be used to disaggregate the population, as demand information becomes available.

5. Conclusions
The main aim of this paper has been to explore an extension of behavioural microsimulation modelling so that third round effects of a policy change can be simulated; these allow for endogenous changes to the distribution of wage rates resulting from the labour supply responses to tax changes. This has involved the concept of the aggregate supply response schedule which identifies the extent to which average labour supply responds to a proportional change in wage rates. Further disaggregation, by using supply response schedules for particular demographic or education groups, provides the possibility of introducing an endogenous change to the form of the distribution of wage rates as well as shifts, since different groups may experience different types of labour supply response to a given tax change, and therefore experience different endogenous wage rate changes.

The use of the concept of the aggregate supply response schedule was illustrated using a behavioural microsimulation model of the Australian economy (MITTS). An increase in the lowest marginal income tax rate was found to reduce labour supply in the second round and consequently raises wages. A range of assumptions was used regarding the pattern of aggregate demand. After rerunning MITTS with a suitable modification to individuals’ wage rates, the third-round effects were found to generate substantially
larger increases in income tax revenue than were suggested by initial non-behavioural costings. The results demonstrated the potential importance of allowing for such third round effects in microsimulation.

References