Employment, Fiscal Policy, and Oligopsonistic Labour Market

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Abstract

This paper presents an investigation of the dynamic effects of fiscal policy in an inter-temporal optimisation model with an oligopsonistic labour market. In an oligopsonistic labour market, fiscal policy expands employment through a shift of labour demand and supply curves. Fiscal policy has a positive effect on physical capital. Consequently, fiscal expansion stimulates not only employment but also aggregate output. However, an increase in government spending that does not generate benefits cannot improve welfare. The extended model provides conditions for justification of active fiscal policy.

1. Introduction

This paper presents an inter-temporal optimisation model with an oligopsonistic labour market to investigate the dynamic effects of fiscal policy. The term ‘oligopsony’ is applicable to any model in which demanders face upward-sloping factor supply. Oligopsony in the labour market is a representative example. Indeed, results of some empirical analyses suggest that employers have considerable market power (see Machin and Manning, 2002; and Manning, 2003). However, few dynamic analyses have examined fiscal policy under oligopsony in the labour market.

2 At the occupational level, a labour market with a high degree of occupational concentration might be recognized as an oligopsonistic market in the sense of industrial organization. Michaelides (2009) calculates the Herfindahl index of local labour markets by occupation. According to his calculation, there is more than one oligopsonistic local market in occupation groups: nursing occupations, teachers, agricultural workers, production occupations, and so on.
3 Manning (2003) explains the source of monopsony power; ignorance among workers about labour market opportunities, individual heterogeneity in preference over jobs, mobility costs, and so on (pp.361-362)
4 Some reports describe labor market oligopsony: Boal and Ransom (1997) survey the literature related to oligopsony power in the labor market. More recently, Bhaskar and To (1999) describe a static model under oligopsony in the labor market.

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We present a dynamic analysis of fiscal policy effectiveness. A departure from
the assumption of a competitive labour market brings about a positive output effect
of fiscal policy without the assumption of productive government expenditure. Under
imperfect competition in the goods market, some investigations of the effectiveness
of fiscal policy derive similar implications (e.g. Mankiw, 1988; Startz, 1989; Heijdra
and van der Ploeg, 1996; and Chen et al., 2005). The mechanism underlying the
results presented herein differs from those described in prior reports in terms of
labour market analysis, although its goals and implications resemble those described
in earlier reports.

In previous studies (monopolistic competition models), fiscal policy is shown
to affect employment through an increase in aggregate demand and crowding out of
private consumption. In contrast, fiscal policy affects output through a shift of labour
supply and demand curves in our model of oligopsony in the labour market. The end
of fiscal policy is the same, but not with respect to the ripple mechanism of policy
change. This paper sheds light on a new channel of fiscal policy effectiveness.

We also present two models of simple extension of the basic model without
changing its details. Analysing these two extended models, it is apparent that the use
of fiscal policy is justified if government spending can compensate for negative welfare
effects arising from the expansion of employment. As described herein, productive
government expenditure has positive effects on both consumption and employment
through extension of the production possibility set. Moreover, the optimal government
size for provision and production of public services differs greatly from that in a
competitive economy.

The remainder of this paper is organised as follows. Section 2 presents
a description of the basic model, which is adaptable to various analyses. Section 3
explains the transitional dynamics and describes analyses of the macroeconomic and
welfare effects of fiscal policy. Section 4 includes descriptions of two models as simple
extensions of a basic model. Section 5 provides some relevant discussion. Finally,
Section 6 presents conclusions of this paper.

2. Basic Model

Households

The representative household is assumed to exist infinitely and to supply labour
elastically. The population of households is normalized to unity. No population growth
exists. The lifetime utility function of the household is formulated as:

\[ U = \int_0^\infty \left[ \ln C(t) - \frac{L(t)^{1+\gamma}}{1+\chi} \right] \exp(-\rho t) dt, \]  

where \( C(t) \) signifies the consumption, \( L(t) \) denotes the labour supply, \( \rho \) stands for the
subjective discount rate, and \( \chi > 0 \). In addition, the budget constraint of a household
is written as:

\[ \dot{K}(t) = r(t)K(t) + \omega(t)L(t) + \Pi(t) - C(t) - T(t), \]  

See Matsuyama (1995) for a general review of these models with monopolistic competition.
where $K(t)$ represents a household’s total assets, $r(t)$ signifies the interest rate, $w(t)$ signifies the wage rate, $\Pi(t)$ denotes the profit dividend, and $T(t)$ stands for the lump-sum tax.\(^6\)

Each household maximises its lifetime utility (1) subject to its budget constraint (2). Solving the optimisation problem confronted by households, the optimality conditions are:

$$\dot{C}(t) = [r(t) - \rho] C(t), \quad \text{(3)}$$

$$L(t) = \left[ \frac{w(t)}{C(t)} \right]^{1+\rho}, \quad \text{(4)}$$

and the transversality condition.

**Firms**

A continuum of firms exists. The total mass of firms is normalised to unity. For firm $i$ final good $Y_i(t)$ is producible using physical capital input $K_i(t)$ and labour input $L_i(t)$. The production function is:

$$Y_i(t) = AK_i(t)^a L_i(t)^{1-a}, \quad \text{(5)}$$

where $A > 0$ and $0 < \alpha < 1$.

The profit of firm $i$ is given as presented below.

$$\pi_i(t) = Y_i(t) - r(t)K_i(t) - w(t)L_i(t) \quad \text{(6)}$$

$$= AK_i(t)^a L_i(t)^{1-a} - r(t)K_i(t) - w(t)L_i(t).$$

Because firms procure physical capital through the competitive capital market, the first order condition of firm $i$ with respect to capital (taking $r(t)$ as given) is:

$$r(t) = \frac{\partial Y_i(t)}{\partial K_i(t)} = AK_i(t)^a a L_i(t)^{1-a-1}, \quad \text{(7)}$$

where $K_i(t) \equiv K_i(t)/L_i(t)$. Using equation (7), profits (6) can be expressed as

$$\pi_i(t) = (1 - \alpha) AK_i(t)^a L_i(t) - w(t)L_i(t) \quad \text{(8)}$$

We assume that the labour market is oligopsonistic. Therefore, firm $i$ chooses $L_i(t)$ to maximise its profit (8) subject to (4), taking $k_i(t)$ as given. The profit maximisation condition for firm $i$ is:

$$\frac{\partial \pi_i(t)}{\partial L_i(t)} = (1 - \alpha)AK_i(t)^a a k_i(t) - \frac{\partial w(t)}{\partial L_i(t)} L_i(t) - w(t) = 0. \quad \text{(9)}$$

Using equation (9) and a symmetric condition of firm, the quantity of employed labour is:

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\(^6\) The profit dividend can be interpreted as factor income aside from capital and labour income (e.g. land).
where \[ q = \left( \frac{1 - \alpha}{1 - \chi} \right) \left( \frac{\Delta K(t)^{\alpha}}{C(t)} \right) = \theta \cdot \left[ \frac{\Delta K(t)^{\alpha}}{C(t)} \right]^{1/\alpha + \chi} \] (10).

In addition, equations (5) and (10) and the symmetric condition yield the aggregate output of:

\[ Y(t) = \theta^{1-\alpha} A^{\alpha + \chi} K(t)^{\alpha + \chi} C(t)^{\alpha + \chi} \equiv Y(C(t), K(t)), \] (11)

where \[ \frac{\partial Y(C(t), K(t))}{\partial C(t)} < 0 \] and \[ \frac{\partial Y(C(t), K(t))}{\partial K(t)} > 0. \]

Finally, using equations (7) and (10) and the symmetric condition, the interest rate is

\[ r(t) = \alpha \theta^{1-\alpha} A^{\alpha + \chi} K(t)^{\alpha + \chi} C(t)^{\alpha + \chi} \equiv r(C(t), K(t)), \] (12)

where \[ \frac{\partial r(C(t), K(t))}{\partial C(t)} < 0 \] and \[ \frac{\partial r(C(t), K(t))}{\partial K(t)} < 0. \]

**Government**

A lump-sum tax is imposed on households to raise for government revenues. Final goods are purchased using tax revenues, as:

\[ G(t) = T(t), \] (13)

where \( G(t) \) signifies government spending.

### 3. Dynamic Analysis

This section characterizes the dynamic properties of the basic model: an investigation of the dynamic system of the economy, and the macroeconomic and welfare effects of fiscal policy.

**Dynamic System and Transitional Dynamics**

Using equations (2), (3), (6), and (11)–(13), the dynamic system can be expressed as:

\[ \dot{C} = \{ r(C(t), K) - \rho \} C, \] (14)

\[ \dot{K} = K(C(t), K) - C - G. \] (15)

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7 In the remainder of this paper, we omit the time index \( t \) from time-variant variables, except for those cases in which it should be called to the reader’s attention (i.e. \( C \) is used as \( C(t) \)). It is noteworthy that \( C(0) \) denotes the initial value of \( C(t) \).
Regarding the existence, uniqueness, and stability of the stationary equilibrium, the following proposition is established (see Appendix for proof of Proposition 1).

**Proposition 1** There exists a unique stationary equilibrium that is stable in the saddle-point sense.

The stationary equilibrium is presented in figure 1. The intersection between \( C \) and \( K \) nullclines is a stationary equilibrium point. There exists a unique stable arm \( SS \) that has a positive slope in \( C-K \) space. The stable arm is characterised by the general solutions of the dynamic system, as:

\[
C(t) = C^* + \Delta \cdot [K(0) - K^*] \exp(\lambda t),
\]

\[
K(t) = K^* + [K(0) - K^*] \exp(\lambda t),
\]

where \( \Delta > 0 \) is the slope of a unique stable arm and \( \lambda < 0 \) is the negative eigenvalue. When \( K(0) < K^*(K(0) > K^*) \), the consumption \( C \) and physical capital stock \( K \) increase (decrease) over time for \( 0 < t < \infty \). They finally converge to their long-run values.

**Dynamic Effects of Fiscal Policy**

We begin our analysis by investigating the long-run effects of fiscal policy on consumption and capital stock. Total differentiation of a stationary dynamic system engenders:

\[
\frac{dC^*}{dG} = \frac{\partial r}{\partial K} \cdot \frac{C^*}{\text{det}J} < 0,
\]

Superscript ‘*’ denotes the stationary value of endogenous economic variables.
\[
\frac{dK^*}{dG} = \frac{\partial r}{\partial C} \cdot \frac{C^*}{\det J} < 0, \tag{19}
\]

where \( \det J < 0 \). Using equations (16)-(19), we can infer the dynamic effects of fiscal policy on \( C \) and \( K \) as:

\[
\frac{dC(t)}{dG} = \frac{dC^*}{dG} - \Delta \cdot \frac{dK^*}{dG} \exp(\lambda t) \leq 0, \tag{20}
\]

\[
\frac{dK(t)}{dG} = [1 - \exp(\lambda t)] \frac{dK^*}{dG} \geq 0. \tag{21}
\]

Figure 2 - Dynamic Effects of Policy Shock

The interpretation of (18)-(21) is explained as follows: Presume that the economy is initially at stationary equilibrium point \( P \) on the stable arm \( SS \) in figure 2, and that a permanent increase exists in government expenditure \( G \). The \( K \) nullcline moves downward. Therefore, the stationary equilibrium point \( P \) shifts to the new point \( P' \) and stable arm \( SS \) also shifts to the new locus \( S'S' \). Figure 2 portrays that \( C \) decreases initially and increases gradually (for \( 0 < t < \infty \)); it then finally converges to new equilibrium point \( P' \). In contrast, \( K \) increases monotonically over time (\( 0 < t < \infty \)) and converges to new equilibrium point \( P' \).

Using equations (10), (11), and (18)-(21), we obtain the dynamic effects of fiscal policy on employment and aggregate output:

\[
\frac{G}{L(t)} \frac{dL(t)}{dG} = \frac{\alpha}{\alpha + \chi} \frac{G}{K(t)} \frac{dK(t)}{dG} - \frac{1}{\alpha + \chi} \frac{G}{C(i)} \frac{dC(i)}{dG} > 0, \tag{22}
\]

\[
\frac{G}{Y(t)} \frac{dY(t)}{dG} = \frac{(1 + \chi)\alpha}{\alpha + \chi} \frac{G}{K(t)} \frac{dK(t)}{dG} - \frac{1 - \alpha}{\alpha + \chi} \frac{G}{C(i)} \frac{dC(i)}{dG} > 0. \tag{23}
\]

\(^9\) See the Appendix for derivation of equations (18) and (19).
The mechanisms of equations (22) and (23) are interpreted as follows. Assume that the initial economy is at equilibrium point P (figure 3). An increase in government spending reduces consumption. Its reduction shifts the labour supply curve downward (from $L_s$ to $L_s'$) and shifts the initial equilibrium point P to the new equilibrium point $P'$ (but it is a temporal equilibrium). Therefore, the firms can afford to hire additional workers because decreased consumption is possible for profit maximisation to expand employment. Indeed, employers incorporate it into their plan. The labour demand $L_d'$ moves to a new locus $L_d''$. Consequently, equilibrium point $P'$ shifts to new equilibrium point $P''$. Furthermore, a decrease in consumption enhances the accumulation of physical capital (figure 2). The resultant higher levels of both capital stock and labour input increase the aggregate output.

The above results can be summarised as follows.

**Proposition 2** In either the short run or long run, a permanent increase in government spending has positive effects on the capital stock, employment, and aggregate output, although it has a negative effect on consumption.
Proposition 2 implies that the impact of fiscal expansion on aggregate output will be positive. However, we turn our analysis to whether the positive effect of fiscal policy on aggregate output improves a household’s welfare. It is convenient for welfare analysis to investigate the effect of fiscal policy on instantaneous utility, such as:

\[ u(t) = \ln C(t) - \frac{L(t)^{1+x}}{1 + \chi}. \]

Differentiating the equation portrayed above with respect to \( G \) and using equations (21) and (22), we obtain:

\[
\frac{du(t)}{dG} = C(t)^{-1} \frac{dC(t)}{dG} - L(t)^{x} \frac{dL(t)}{dG} \\
= \left[ 1 + \frac{L(t)^{1+x}}{\chi} \right] C(t)^{-1} \frac{dC(t)}{dG} - \frac{\alpha}{\chi} L(t)^{1+x} \frac{dK(t)}{dG} < 0.
\]

Although fiscal expansion engenders increased output through positive effects on capital stock and employment, it simultaneously reduces consumption and raises the disutility of the labour supply. For that reason, an increase in government spending is harmful to a household’s welfare.

Regarding the welfare effect of fiscal policy, the following proposition is established.

**Proposition 3** A permanent increase (decrease) in government spending raises the welfare loss (gain).

### 4. Further Analyses

The result described in the preceding section is based on the assumption that government spending generates no benefits. However, including consideration of the viewpoint of public economics, fiscal policy might be able not only to increase aggregate output but also to improve welfare. In this section, considering the direct and indirect benefits of fiscal policy, we investigate the effects of fiscal policy.

**Government Spending in the Utility Function**

One simple way to incorporate \( G \) in the utility function is the case in which private consumption and government purchases are substitutes. Then, the lifetime utility function is:

\[
U = \int_{0}^{\infty} \left[ \ln (C_p + \gamma G) - \frac{L(t)^{1+x}}{1 + \chi} \right] \exp(-\rho t) dt, 
\]

where \( C_p \) denotes private consumption, and \( \gamma \geq 0 \). If \( C \equiv C_p + \gamma G \) is defined, then the household budget constraint is:

\[
\dot{K} = Y(C, K) - C - (1 - \gamma) G.
\]
All properties of the basic model are still applicable if $0 \leq \gamma < 1$. Therefore, the welfare-maximizing policy is that of shutting down of government, although government spending has a positive effect on output. If complementarity exists between private consumption and government purchases, then government spending might be welfare-improving. However, its complementarity negatively affects both employment and output.

**Government Spending in the Production Function**

In the analyses presented in this subsection, government spending is incorporated into the production function as productive government expenditure. We assume that the productivity depends on productive government expenditure to maintain the basic setting, as:

$$A = G^\beta,$$

(27)

where $0 < \beta < 1$. Then, equations (10)-(12) are rewritten as:

$$L = \theta \cdot \left[ \frac{AG^\beta K^\alpha}{C} \right]^{1/(\alpha + \chi)},$$

(28)

$$Y = \theta^{1-x} G^{\frac{1}{\alpha + \chi}} K^{\frac{1}{\alpha + \chi}} C^{-\frac{1}{\alpha + \chi}} \equiv Y(C, K, G),$$

(29)

$$Y = \alpha \theta^{1-x} G^{\frac{1}{\alpha + \chi}} K^{\frac{1}{\alpha + \chi}} C^{-\frac{1}{\alpha + \chi}} \equiv r(C, K, G).$$

(30)

As presented above, we assume that $(1 + \chi) \beta(\alpha + \chi) < 1$. All of $L$, $Y$, and $r$ are increasing in $G$. The modified Kaizuka condition implies the optimal size of government as $G/Y = (1 + \chi) \beta(\alpha + \chi) > \beta$.

Using equations (2), (3), (6), (13), (29), and (30), the dynamic system can be expressed as:

$$\dot{C} = [r(C, K, G) - \rho]C,$$

(31)

$$\dot{K} = Y(C, K, G) - C - G.$$

(32)

Equations (31) and (32) are nearly equivalent to the dynamic system of equations (14) and (15) (i.e. Proposition 1 is still pertinent).

The long-run effects of fiscal policy on $C$ and $K$ are given as follows (see the Appendix for derivation of equations (33) and (36)):

$$\frac{dC^*}{dG} = \left( \frac{\partial C^*}{\partial G} \right)_t + \left( \frac{\partial C^*}{\partial G} \right)_b \approx 0.$$

(33)

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In those equations,
\[
\left( \frac{\partial C^*}{\partial G} \right)_l - \frac{\partial r}{\partial K} \cdot \frac{C^*}{\det J} < 0,
\]
(34)
\[
\left( \frac{\partial C^*}{\partial G} \right)_p = \left[ \frac{\partial Y}{\partial G} \frac{\partial r}{\partial K} - \frac{\partial r}{\partial G} \frac{\partial Y}{\partial K} \right] \cdot \frac{C^*}{\det J} > 0.
\]
(35)

Actually, equation (34) corresponds to equation (14) (the effects of fiscal policy without a positive productivity effect: an indirect effect). Equation (35) is the positive productivity effect of fiscal policy (direct effect). Equation (33) is positive (i.e. \( \frac{dY}{dG} > 0 \)) if \( \frac{\partial Y}{\partial G} \leq 1 \). Furthermore, we have:
\[
\frac{\partial K^*}{\partial G} = \left( \frac{\partial K^*}{\partial G} \right)_l + \left( \frac{\partial K^*}{\partial G} \right)_p \not\equiv 0,
\]
(36)
where
\[
\left( \frac{\partial K^*}{\partial G} \right)_l = \frac{\partial r}{\partial C} \cdot \frac{C^*}{\det J} > 0,
\]
(37)
\[
\left( \frac{\partial K^*}{\partial G} \right)_p = -\frac{\partial r}{\partial G} \cdot \frac{C^*}{\det J} > 0.
\]
(38)

Actually, equation (37) is fundamentally equivalent to equation (19). Equation (38) represents the positive productivity effect of fiscal policy. The presence of a direct effect of public input enhances the positive effect on consumption and capital accumulation.

The long-run effect of fiscal policy on employment is:
\[
\frac{G}{L^*} \frac{\partial L^*}{\partial G} = \frac{\beta}{\alpha + \chi} + \frac{\alpha}{\alpha + \chi} \frac{G}{K^*} \frac{dK^*}{dG} - \frac{1}{\alpha + \chi} \frac{G}{C^*} \frac{dC^*}{dG}
\]
\[
= \frac{\beta}{\alpha + \chi} + \frac{\alpha}{\alpha + \chi} \left[ \left( \frac{\partial K^*}{\partial G} \right)_l + \left( \frac{\partial K^*}{\partial G} \right)_p \right] - \frac{1}{\alpha + \chi} \left[ \left( \frac{\partial C^*}{\partial G} \right)_l + \left( \frac{\partial C^*}{\partial G} \right)_p \right] \not\equiv 0.
\]
(39)

Equation (39) is ambiguous. Fiscal expansion has a positive effect on employment if the direct effect of fiscal policy on consumption is sufficiently smaller than the weighted sum of other effects. Then, aggregate output is also increasing in government expenditure. In cases where government spending is in the production function, it might be possible for government to improve not only the level of employment and aggregate output, but also that of welfare. Thereby, the optimal government size differs greatly from that in a competitive economy.

5. Discussion

Labour Market Imperfection and Goods Market Imperfection

Some results presented in section 3 and 4, especially long-run results, resemble those of existing studies of monopolistic competition of goods market and fiscal policy effectiveness (e.g. Mankiw, 1988; Startz, 1989; Matsuyama, 1995; and Reinhorn, 1998). Their results are generally interpreted as follows.\(^\text{12}\)}
The profits of firms and the consumer’s income are mutually reinforcing because of the existence of distortion that sets a price above marginal cost. Greater demand for goods gives rise to larger profits and consumer income; moreover, larger income feeds back demand for goods through consumption. Thereby, fiscal expansion generates a multiplier effect. In a dynamic model of uncompetitive goods market (by Costa and Dixon, 2009), the underlying mechanism is the same as that explained above.

In contrast, as stated previously, the source of a positive output effect of fiscal policy in our study is based mainly on the labour market’s consequence. To verify this point, we now compare the mechanism of our model with that of Costa and Dixon (2009) using figure 3 and figure 4 for illustration. The BC line (gray line in figures 3 and 4) is the household budget constraint, the EF line (black line in figures 3 and 4) is an Euler frontier, and the IEP curve is an income expansion path.\footnote{Reinhorn (1998) shows that optimal fiscal expenditure is equal to zero in the model developed by Mankiw (1988).} \footnote{This explanation is developed by Matsuyama (1995) and Costa and Dixon (2009).}\footnote{Euler frontier stands for a steady-state relation between income and consumption (derived from (2); see Costa and Dixon (2009) for details). The income expansion path corresponds to the optimality condition for consumption and labour with stationary wage level.}

Figure 4 - Fiscal Policy Shock Under Goods Market Imperfection
Effect of fiscal policy under an oligopsonistic labour market (Reconsideration):
Under a balanced budget regime, an increase in government spending is equivalent to an increase in the lump-sum tax. An increase in the lump-sum tax decreases the disposable income of a household. The line EF moves to EF′ at figure 3. This shift and optimal condition for consumption and labour (IEP) give the downward shift of the labour supply curve (Lsg → L′s in figure 3). In response to the downward shift of the labour supply curve, firms increasingly offer labour demand (because they know the labour supply function). Consequently, in equilibrium, an increase in government spending engenders a decrease in consumption (i.e., an increase in physical capital) and an increase in the labour supply. Therefore, the aggregate output is also increased by an increase in government spending.

Effect of fiscal policy under a competitive labour market (with monopolistic competition of goods market):
An increase in government spending raises aggregate demand. An expansion of aggregate demand increases the required labour input for consumption size C0 → L0 → L1. To supply L1 units of labour, the consumption size should be decreased according to IEP. A decrease in consumption pulls down aggregate demand: government spending crowds out private consumption. Consequently, in equilibrium, the shift of the labour supply curve is Lsg → L′s. Therefore, the effective increase in the labour supply is L′ – L0. An increase in government spending increases the aggregate output.

Common and different points: The model of oligopsony in the labour market and of monopolistic competition in the goods market shows the same result: an increase in government spending. This is a common point. A different point is a ripple mechanism of fiscal policy. In the model of oligopsony in the labour market, fiscal policy affects output through the labour supply and demand shift (labour market) although it affects employment through increased aggregate demand and crowding out of private consumption. We clarify a new channel of fiscal policy effectiveness. Twisting the market imperfection of these two different types might provide new insight into the effectiveness of fiscal policy.

Trade Unions
In an oligopsonistic labour market, firms choose their demand for labour subject to the labour supply of households. That contrasts to a labour market with trade unions, which maximise their objective function subject to the labour demand schedule. Models of trade unions consist of an efficient bargain model (e.g. McDonald and Solow, 1981; and Grout, 1984) and a right-to-manage model (e.g. Van der Ploeg, 1987). Discussion of trade unions and an oligopsonistic labour market is beneficial for further analyses.

Although models of trade unions have many noteworthy points, the most notable is the level of the union-determined wage. Normally, the union wage is higher than the market wage and an excess supply exists for labour. Therefore, as is often the case with these models, trade unions can be regarded as job destroyers. However, the oligopsonistic labour market, in which the firm as employer has considerable market

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14 Grout (1984) examines the effects of trade unions on capital, and treats the hold-up problem. Hart and Moore (1988) show that underinvestment is a result of non-binding contracts in general settings.

15 This result depends on the union’s preferences.
power, will bring about a revolutionary change in the role of the trade union. Indeed, trade unions in an oligopsonistic labour market will raise wages for workers; the union wage will exceed the firm-determined wage. Trade unions will serve as guardians for workers. Consequently, the analysis of the effect of trade union or unionisation of workers should be developed in an oligopsonistic labour market.

**Inter-temporal Maximisation of Firm Profits**

In previous sections, firms’ optimisation problems are static ones. What difference arises if firms explicitly optimize in an inter-temporal fashion? We next investigate firm decision-making in an inter-temporal fashion. The objective of a firm is maximizing the net present value of future profits $V_i$. The firm chooses its investment size and employment size to maximise $V_i$ subject to (3) and (4), and the accumulation equation of physical capital.

Formally, we establish the following.

$$
\max_{l(t), i(t)} V_i = \int_0^\infty \left[ AK(t)^{\alpha} L(t)^{1-\alpha} - I(t) - w(t)L(t) \right] \exp \left( - \int_0^t r(s) ds \right) dt.
$$

This equation is subject to $K_i(t) = I_i(t), (3), (4), K_i(0)$, and $\{ r(t) \}_{t=0}^\infty$. Inter-temporal optimisation engenders (7) and (9). Without dynamic friction (e.g. installation costs of physical capital or labour) or market power in the capital market (e.g. controlling $r(t)$) or in the goods market (e.g. monopolistic competition), the firm’s inter-temporal optimisation is fundamentally equivalent to a static one.

### 6. Conclusion

This paper developed an inter-temporal optimization model with an oligopsonistic labour market to examine the dynamic effects of fiscal policy. In the oligopsonistic labour market, the labour demand is inelastic with respect to the real wage. Fiscal policy shifts the labour supply curve downward and it brings about a temporary decrease in the real wage. After such a decrease in the labour price, labour demand expands because a profit-maximising employer can afford to hire additional workers (because employers incorporate the labour supply schedule into their production plan). Furthermore, physical capital accumulation follows expansion of employment. Consequently, fiscal expansion promotes employment and increases aggregate output.

However, the results also show that fiscal expansion such as an increase in non-beneficial government spending is harmful to social welfare. The positive output effect of an increase in non-beneficial government spending alone is insufficient to justify the use of fiscal policy. Based on our analysis, a sufficient condition for justifying the use of fiscal policy is the determination of what fiscal policy brings about not only employment expansion but also consumption expansion. Productive government expenditure satisfies that sufficient condition because that expenditure extends the production possibility set.

Finally, we consider the direction of future research. First, adopting an alternative type of competition in an oligopsonistic labour market yields important insights. As described herein, we assume that Cournot competition exists in the
labour market. However, Cournot competition might be insufficient to explain the wage distribution in a real labour market. Labour market competition of another type (e.g. Bertrand competition) will compensate for a limitation of explanation of the wage distribution. Second, it is interesting to investigate various financial sources of government spending. The dynamic effects of fiscal policy differ slightly from those of this paper if a distortionary tax, such as a factor income tax, is adopted by government to finance government spending. These topics present important avenues for future investigations.

Appendix

A.1. Proof of Proposition 1

The nullclines of the system are

\[ 0 = r(C, K) - \rho \dot{C} = 0, \quad (40) \]
\[ 0 = Y(C, K) - C - G \dot{K} = 0. \quad (41) \]

Using equation (40), the C nullcline can be shown to have the following properties:

\[ \frac{dC}{dK} = - \frac{\partial r}{\partial K} \frac{dr}{\partial C} < 0, \quad \lim_{K \to 0} C = + \infty, \quad \text{and} \quad \lim_{K \to +\infty} C = 0. \quad (42) \]

From equation (41), the K nullcline is inferred to have the following properties:

\[ \frac{dC}{dK} = - \frac{\partial Y}{\partial K} \left( 1 - \frac{\partial Y}{\partial C} \right) > 0, \quad \lim_{K \to 0} C = 0, \quad \text{and} \quad \lim_{K \to +\infty} C = + \infty. \quad (43) \]

Equations (42) and (43) portray the existence of a unique intersection point of all nullclines.

The linearised system is

\[
\begin{pmatrix}
\dot{C} \\
\dot{K}
\end{pmatrix} =
\begin{pmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{pmatrix}
\begin{pmatrix}
C - C^* \\
K - K^*
\end{pmatrix},
\quad (44)
\]

where

\[ J_{11} = - \frac{\partial r}{\partial C} C^* < 0, \quad J_{12} = \frac{\partial r}{\partial K} C^* < 0, \quad J_{21} = \frac{\partial Y}{\partial C} - 1 < 0, \quad \text{and} \quad J_{22} = \frac{\partial Y}{\partial C} > 0. \]

The determinant of the coefficient matrix of equation (44) is

\[ \det J = J_{11}J_{22} - J_{12}J_{21} = \odot + - \odot < 0. \]

Consequently, the stationary equilibrium is saddle-point stable.

Dunlop (1957) and Groshen (1991) report the empirical evidence of the wage distribution.
A.2. Derivation of Equations (16) and (17)

General solutions of the linear system given as equation (44) are

\[ C(t) - C^* = A_{11} \exp(\lambda t) + A_{12} \exp(\mu t), \]  \hspace{1cm} (45)

\[ K(t) - K^* = A_{21} \exp(\lambda t) + A_{22} \exp(\mu t), \]  \hspace{1cm} (46)

where \( A_{ij} \) signifies the vector for arbitrary constants \((i, j = 1, 2)\), \( \lambda \) denotes the negative eigenvalue, and \( \mu \) stands for the positive eigenvalue. Because \( K(0) \) are not jumpable, we have \( A_{22} = 0 \).

Inserting \( A_{22} = 0 \) into equation (46) and differentiating that equation with respect to time yields

\[ \dot{K}(t) = A_{21} \exp(\lambda t). \]  \hspace{1cm} (47)

Subsequently, using \( A_{22} = 0 \) along with equations (44)-(46), we obtain

\[ \dot{K}(t) = J_{21} \cdot \left[ A_{11} \exp(\lambda t) + A_{12} \exp(\mu t) \right] + J_{22} A_{21} \exp(\lambda t). \] \hspace{1cm} (48)

Combining equation (47) with equation (48), the vector \( A_{ij} \) is expected to satisfy these equations:

\[ A_{12} = A_{22} = 0, \] \hspace{1cm} (49)

\[ (\lambda - J_{22})A_{21} = J_{21} A_{11}. \] \hspace{1cm} (50)

At time \( t = 0 \), Eqs. (46) and (49) yield \( A_{21} = (K(0) - K^*). \)

Substituting \((K(0) - K^*)\) for \( A_{21} \) in equation (46), we obtain

\[ K(t) = (K(0) - K^*) \cdot \exp(\lambda t), \] \hspace{1cm} (51)

and insertion of \( A_{21} = (K(0) - K^*) \) into equation (50) gives

\[ A_{11} = \frac{(\lambda - J_{22})(K(0) - K^*)}{J_{21}}. \] \hspace{1cm} (52)

Using equations (45), (49), and (52), we arrive at

\[ C(t) - C^* = \frac{(\lambda - J_{22})(K(0) - K^*)}{J_{21}} \cdot \exp(\lambda t). \] \hspace{1cm} (53)

Equations (51) and (53) yield (16) and (17).
A.3. Derivation of Equations (18) and (19)

Total differentiation of the stationary system of equations (14) and (15) yields

\[
\begin{pmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{pmatrix}
\begin{pmatrix}
dC^* \\
dK^*
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
dG
\end{pmatrix}, \tag{54}
\]

Applying Cramer’s rule to equation (54), we obtain

\[
dC^* = - \frac{J_{12}}{\det J} \cdot \frac{\partial r}{\partial K} \cdot C^* < 0,
\]

\[
dK^* = \frac{J_{11}}{\det J} \cdot \frac{\partial r}{\partial C} \cdot C^* < 0.
\]

A.4. Derivations of Equations (33) and (36)

Total differentiation of a stationary system of equations (31) and (32) yields

\[
\begin{pmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{pmatrix}
\begin{pmatrix}
dC^* \\
dK^*
\end{pmatrix}
= 
\begin{pmatrix}
- \frac{\partial r}{\partial C} C^* dG \\
- \frac{\partial r}{\partial G} dG + dG
\end{pmatrix}, \tag{55}
\]

Applying Cramer’s rule to equation (55), we obtain

\[
dC^* = \left[ \frac{\partial Y}{\partial G} - 1 \right] \frac{\partial r}{\partial K} \cdot C^* \frac{\partial C^*}{\partial G} + \left( \frac{\partial C^*}{\partial G} \right)_b \approx 0,
\]

\[
dK^* = \left[ \frac{\partial r}{\partial C} \right] \cdot C^* \frac{\partial K^*}{\partial G} + \left( \frac{\partial K^*}{\partial G} \right)_b \approx 0.
\]

References


