Policy Evaluation, Welfare Weights and Value Judgements: A Reminder

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Abstract

This paper is concerned with the use of social welfare functions in evaluating changes. In particular, it considers suggestions that welfare weights to be used in comparing the gains and losses of different individuals (or other appropriate units of analysis), and a social time preference rate for use in cost benefit evaluation, can be estimated either from consumers’ behaviour or from the judgements implicit in tax policy. It is suggested that results are highly sensitive to the context and model specification assumed. More importantly, the argument that an estimated elasticity of marginal utility or time preference rate should be used in policy evaluations fails to recognise that fundamental value judgements are involved. Various estimates may be of interest, but they cannot be used by economists to impose value judgements. The main contribution economists can make is to examine the implications of adopting a range of alternative value judgements.

1. Introduction

The aim of this paper is to review and critically examine a number of frameworks in which the concept of the ‘elasticity of marginal valuation’, in the context of evaluating a social welfare function, is central. This arises particularly in assigning welfare weights in inequality comparisons and in cost-benefit analyses involving discounting, for example in discussions of population ageing or environmental policy. Stress is placed on the need to distinguish these contexts and models clearly, in order to avoid the possible inappropriate ‘transfer’ of a value from one context to another. It is argued that this central elasticity concept cannot in fact be measured objectively but necessarily involves value judgements. Hence the role of economists is not to propose the use of particular values, which is equivalent to imposing judgements, but to examine the implications of adopting alternative value judgements. The view put forward here is in line with – and is indeed intended as a reminder of – that of Robbins (1935, p.148) when he famously argued, ‘between the generalisations of positive and normative studies there is a logical gulf fixed which no ingenuity can disguise and no juxtaposition in space or time bridge over’.

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Social welfare functions are used both in static contexts, involving a distribution over appropriately defined units, and in dynamic contexts, involving a distribution over time periods. Decisions must be made regarding the ‘welfare metric’ (for example, income, consumption, or utility) and the unit of analysis (for example, households, individuals or equivalent adults). Both of these decisions also involve value judgements, but they are not the concern of the present paper. The emphasis here is on the fact that, in attaching ‘welfare weights’ to each unit or time period, the concept of the elasticity of the marginal valuation of the chosen welfare metric plays an important role. For example, it is prominent in the current fierce debate on the economics of climate change: see, for example, Stern (2006) and criticisms of the discount rate used by Stern which include Nordhaus (2006), Dasgupta (2006) and Carter et al. (2006).

Many contributions to the literature appear to regard this elasticity as something that can be objectively measured. For examples of the use of various approaches to ‘measuring’ this elasticity, extensive references and a range of ‘estimates’, see Pearce and Ulph (1998), Cowell and Gardiner (1999) and Evans (2005). In fact, the UK Treasury recommends a value for the elasticity of around unity, based on empirical estimates. Evans (2005) argued that the Treasury recommendation, when forming distributional welfare weights or computing a social time preference rate in cost-benefit studies, is too low. Pearce and Ulph (1998, p.282) argued that the earlier UK Treasury recommended discount rate at the time was too high, saying that ‘we find it impossible to support the continued use of rates in the region of six per cent for the UK. Such rates are far too high’. In each of these examples, the view was presented as an objective finding, rather than a value judgement.

Section 2 briefly introduces the form of social welfare function and welfare weights widely adopted in the literature on policy evaluation in the single-period context. The concept of the elasticity of marginal valuation is defined and the use of ‘estimates’ based on empirical information about a sample of individuals’ consumption behaviour is criticised. Section 3 turns to the multi-period context and its additional complications. Much emphasis has been placed on the estimation, and adoption, of a value of the elasticity that is thought to be implicit in an income tax structure. Attempts to impute such a value include Mera (1969), Stern (1977), Christiansen and Jansen (1978), Moreh (1981), Brent (1984), Cowell and Gardiner (1999) and Evans (2005). This approach, based on an assumption of equal absolute sacrifice as a policy objective, necessarily produces for a progressive tax a value of the elasticity of marginal valuation in excess of unity. Section 4 considers, first, whether this approach can actually provide reliable evidence of implicit judgements and, second, whether it could legitimately be used as the foundation of an argument in favour of using those values.

Section 5 briefly gives some idea of the implications, in terms of value judgements, of adopting alternative values of the elasticity. This involves the idea, familiar from the literature on inequality measurement, of the ‘leaky bucket’ experiment, which makes explicit the tolerance of losses when making income transfers between individuals. The main conclusions are in section 6, where the fundamental difference between what ‘is’ and what ‘should’ be is again stressed. Given the need to make value judgements, economists have no special qualifications or authority to impose their own judgements on others, and cannot use ‘estimates’ as support for their views.
Economists have an obligation to make value judgements explicit, and their role is to examine the implications of adopting a range of such judgements. Having been presented with alternative results, readers can then form their own opinions. Ultimately, the paper aims to make clear some important distinctions which are often confused in the literature.

2. Social Evaluations: A Single Period

Any attempt to answer the question, ‘when is a change an improvement?’, faces the fundamental difficulty that it cannot avoid the use of value judgements. Hence complete agreement in any particular context – say the effect of a proposed change to a tax and transfer system – is most unlikely, even if there are no losers.\(^1\) The approach adopted in economics is to specify explicit value judgements in a formal manner, using a social evaluation function or, following Samuelson (1947), a ‘Social Welfare Function’. Crucially, this function formally expresses the value judgements of a fictional judge or policy maker. It is not, despite the use of the term ‘social’, intended to represent any kind of aggregate or representative views of society.\(^2\)

Indeed the judge is considered to be an independent person who is not affected by the outcomes. This section discusses the form of social welfare function commonly used and the associated elasticity of marginal valuation.

The Welfare Function

Some simple properties of welfare functions are typically specified, with the hope that while they cannot be expected to represent any kind of consensus, they are at least likely to appeal to a large number of people. It is in this spirit that the form of evaluation function widely adopted reflects adherence to value judgements such as the ‘principle of transfers’ (whereby a transfer from a richer to a poorer person is judged to produce an ‘improvement’\(^3\)) as well as being individualistic, additive and Paretian.\(^4\)

The first choice is to select a welfare metric, say \(x_h\) for unit \(h\), and to define the unit of analysis itself, with both choices involving value judgements. The definition of \(x\), the welfare metric, is problematic and usually depends on the context. It is variously defined as income, consumption, utility or money metric utility, where each concept may, in addition, be expressed in ‘adult equivalent’ terms, and such equivalence scales themselves involve difficult value judgements. The choice of unit of analysis, between for example individuals and adult equivalents, also involves incompatible value judgements. This important issue is not discussed here, and for simplicity the unit is referred to below simply as the individual.\(^5\)

\(^1\) The Pareto criterion has little practical use as it refuses to pass judgement where losers exist, and is certainly not a value-free criterion. There are also well-known problems with the use of ‘potential Pareto improvements’.

\(^2\) The extreme case of a ‘representative agent’, mentioned below, is the exception where the welfare function corresponds to the utility function of the fictional representative.

\(^3\) This is conditional on the transfer being such that the transferee does not become richer than the transferor.

\(^4\) However, other approaches are widely used. For example, the welfare function implicit in the use of the Gini inequality measure involves quite different value judgements.

\(^5\) On these aspects see, for example, Shorrocks (2004). The choice of different units can have quite different implications for policy evaluations; for examples in the single period evaluation of tax reforms see Creedy and Scutella (2004) and in the multi-period case, see Creedy and Guest (2006).
An additive social welfare function is typically formed as an appropriate weighted sum. Thus the problem is to specify precisely how those weights, referred to as ‘welfare weights’, are formed. Define the contribution to social welfare of individual $h$ as $W(x_h)$. Social welfare, $W_S$, is thus defined as:

$$W_S = \sum_{h=1}^{H} W(x_h) \quad (1)$$

Importantly, $W(x_h)$ is not individual $h$’s utility function. Indeed, the welfare metric may itself be utility (or some money metric measure of utility), as in the optimal tax literature. In the marginal indirect tax reform literature it represents indirect utility.

### The Elasticity of Marginal Valuation

Any change - which may be induced by a policy reform to an income or indirect tax structure - gives rise to a total change in social welfare of:

$$dW_S = \sum_{h=1}^{H} \frac{\partial W(x_h)}{\partial x_h} dx_h \quad (2)$$

Letting $v_h = \frac{\partial W(x_h)}{\partial x_h}$ denote the ‘marginal valuation’, the contribution to the change in $W_S$ of a change in $x_h$, (2) becomes:

$$dW_S = \sum_{h=1}^{H} v_h dx_h \quad (3)$$

The term $v_h$ represents the ‘welfare weight’ attached to the $h$th individual. An aversion to inequality on the part of the hypothetical judge is specified by an assumption that $W(.)$ is concave, so that it satisfies the ‘principle of transfers’. A measure of relative inequality aversion, $R$, is therefore based on the concavity measure:

$$R = -\frac{xd^2W(x)/dx^2}{dW(x)/dx} \quad (4)$$

This is equivalent to the ‘elasticity of marginal valuation’, $(\frac{dv}{dx})(\frac{\delta v}{\delta x})$. This elasticity

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6 For ‘classical utilitarians’, the evaluation criterion was simply the sum, over all individuals, of utilities. The welfare weights were thus all unity.

7 In the literature on marginal tax reform, $W$ is defined in terms of (indirect) utilities, $V_h$, so that $W = \sum_{h=1}^{H} W(V_h)$ and the effect of a change in the price of good $i$, $p_i$, say arising from a tax change, is:

$$\frac{\delta W}{\delta p_i} = \sum_{h=1}^{H} \frac{\delta W(V_h)}{\delta V_h} \frac{\delta V_h}{\delta p_i}$$

From Roy’s Identity, $x_{hi} = - (\delta V_h / \delta p_i / \delta V_h / \delta m)$, where $m$ is total expenditure, and:

$$\frac{\delta W}{\delta p_i} = \sum_{h=1}^{H} \frac{\delta W(V_h)}{\delta V_h} \frac{\delta V_h}{\delta m} x_{hi}$$

In specifying the term in brackets, $W$ is usually re-interpreted in terms of total expenditures, with $W(m_h) = m_h^{1-\varepsilon} / (1 - \varepsilon) \varepsilon$ and so the term in brackets becomes $m_h^{\varepsilon}$. 

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plays an important role in what follows. Importantly, the term ‘marginal valuation’ is used here rather than using ‘marginal utility’ since, as stated above, \( W(x) \) is not a utility function. The function \( W(x) \) represents the contribution to social welfare (that is to the social evaluation function) of the \( h \)th person, however \( x \) is defined.

**Choice of Values**

The function \( W(x) \) is specified in the vast majority of studies as:

\[
W(x) = \frac{x^{\epsilon - 1}}{1 - \epsilon},
\]

for \( \epsilon \neq 1 \), and \( W(x) = \log x \), where \( \epsilon = 1 \). In this case, substitution in (4) gives \( R = \epsilon \) and \( \epsilon \) thus reflects a constant degree of relative inequality aversion of the judge or policy maker.

Importantly, \( \epsilon \) is not an objective measure relating to individuals in society, but reflects the subjective value judgements of a fictional judge who is evaluating the effects of alternative policies or outcomes. However, a superficially similar-looking, but very different, concept of the ‘elasticity of marginal utility’ plays a central role in some consumer demand systems, particularly where directly additive utility functions are involved; this was first clarified by Frisch (1959). Define the elasticity of the marginal utility of total expenditure with respect to total expenditure as \( \xi \). If \( \delta_{ij} \) denotes the Kroneker delta, such that \( \delta_{ij} = 0 \) when \( i \neq j \), and \( \delta_{ij} = 1 \) when \( i = j \), and \( e_j \) is the total expenditure elasticity for good \( j \), and \( w_j \) is the budget share of good \( j \), Frisch showed that the price elasticities, \( e_{ij} \), can be written as:

\[
e_{ij} = -e_j w_j \left( 1 + \frac{e_j}{\xi} \right) + \frac{e_i \delta_{ij}}{\xi}
\]

This can be used to obtain \( \xi \), given independent values of the elasticities. In the special case of the Linear Expenditure System, the elasticity of marginal utility has a convenient interpretation: it is the ratio of total expenditure to supernumerary expenditure, that is, expenditure above a ‘committed’ amount. The use of the LES in empirical demand studies therefore necessarily involves an elasticity value of, say \( \epsilon_{LES} \), which is well above unity, and studies typically obtain a value of around 2.

In view of the entirely different contexts, of welfare comparisons involving social evaluation functions and empirical studies of household consumption behaviour, there is no relationship whatsoever between \( \epsilon \) and \( \epsilon_{LES} \). In other words, there is absolutely no reason why a value of \( \epsilon_{LES} \) to be imposed by economists in making comparisons, could be ‘estimated’ using information from studies of household budgets. Nevertheless, this suggestion is sometimes made; for example, elasticities obtained on the basis of the Linear Expenditure System are discussed by Evans (2005, pp. 204-206).

In using \( \epsilon \) values to compute values of social welfare functions, or carry out cost-benefit evaluations, there is no alternative but to consider a range of alternative values, implying different degrees of aversion to inequality. In some cases, ‘dominance’ results may be obtained. In other words one policy may be judged to give rise to a higher value of social welfare than another policy for all values of \( \epsilon \). In other situations, readers can make up their own minds given the reported computations.
In considering alternative values of $\varepsilon_s$, it is clearly useful to ensure that they are within a range that is considered appropriate by a reasonable number of users of the results. Hence questionnaire studies have been designed to elicit information about individuals’ value judgements. It was in this spirit that the questionnaire study of Amiel et al. (1999) was carried out. Nevertheless, there was no suggestion that questionnaires can produce any single value that should be used in policy evaluations.\(^8\) A substantial number of respondents did not adhere to the constant relative inequality aversion form. In addition, Amiel and Cowell (1992, 1994) have found that a large number of questionnaire respondents do not actually share the value judgements that are explicit in the most common forms of social welfare function used in evaluation work, such as the one discussed above. This presents a challenge to produce alternative flexible specifications.\(^9\)

3. Multi-Period Contexts

Evaluations are also made in a multi-period context. Some models use a representative individual, and hence subsection 3.1 begins by looking at a single individual optimisation in a multi-period framework. Subsection 3.2 then considers social evaluation functions.

A Single Individual

Consider a single individual where $C_t$ represents consumption in period $t$. An additive utility function defined over $T$ periods, where $\rho$ is the ‘pure time preference’ rate of the individual is thus:

$$U_T = \sum_{t=1}^{T} \left( \frac{1}{1 + \rho} \right)^{t-1} U(C_t)$$

Consider periods 1 and 2. The pure time preference, or impatience, rate measures the extent to which the slope of an indifference curve, at a point where $C_1 = C_2$, deviates from a downward sloping (from left to right) 45 degree line.\(^{10}\) It reflects impatience, or an ‘aversion to waiting’ on the part of the individual, whereby faced with a constant consumption stream the individual is prepared to give up more than one unit of $C_2$ in order to obtain one more unit of $C_1$. For any combination of $C_1$ and $C_2$, the marginal rate of substitution between consumption in the two periods, $MRS_{C_1,C_2}$, is the absolute value of the slope of the individual’s indifference curve, and is thus:

$$\frac{\partial U_T}{\partial C_1} \frac{\partial C_1}{\partial U_T} \frac{\partial C_2}{\partial U_T}$$

The discount rate, $r$, at any combination of $C_1$ and $C_2$, is defined as:

$$1 + r = MRS_{C_1,C_2}$$
Supposing that \( U(C) \) is the iso-elastic form:

\[
U(C) = \frac{C^{1-\varepsilon_p}}{1-\varepsilon_p}
\]  

(9)

where \( \varepsilon_p \) is the absolute value of the individual’s elasticity of the marginal utility of consumption.\(^{11}\) Then:

\[
MRS_{C_1,C_2} = \left(\frac{C_2^{1-\varepsilon_p}}{C_1^{1-\varepsilon_p}}\right) (1 + \rho)
\]

(10)

Figure 1 – Time Preference

An example is shown in figure 1. At the point of intersection with the 45 degree line from the origin, along which consumption is equal in both periods, the solid indifference curve shown is steeper than the downward sloping 45 degree line, indicating a degree of pure time preference. The convexity of the indifference curve is affected by the value of \( \varepsilon_p \), so that the solid curve reflects a lower value than the broken curve. If \( \varepsilon_p = 0 \), the indifference curves are straight lines and the individual’s optimal position would be a corner solution, consuming everything either in period 1 or 2, depending on whether the market rate of interest (assuming equal borrowing and lending rates) is less than or greater than the pure time preference rate. In general the individual’s optimal position is a tangency where the market rate of interest equals the discount rate.

A convenient expression for the discount rate can be obtained using an

\(^{11}\) It is also the absolute value of the elasticity of the marginal rate of substitution with respect to the ratio of consumption levels.
approximation which holds when the various rates are small. Let $g$ denote the (constant) growth rate of consumption, so that $\frac{C_2}{C_1} = 1 + g$ and:

$$r = (1 + g)^p (1 + \rho) - 1$$  \hfill (11)

Expanding $(1 + g)^p$ and neglecting squared and higher-order powers gives $(1 + g)^p \approx 1 + \varepsilon p g$. Hence $r = (1 + \varepsilon p g) (1 + \rho) - 1$, and making the further assumption that $\varepsilon p g \rho p = 0$, the individual’s discount rate is:

$$r = \rho + \varepsilon p g$$  \hfill (12)

Hence the discount rate is equal to the pure time preference rate plus the product of the growth rate of consumption and the individual’s (absolute) elasticity of the marginal utility of consumption. The growth rate affects the difference between consumption in the present and future, and $\varepsilon p$ reflects an aversion to inequality between periods (not impatience to consume in the present), and the combination of these involves an addition to pure time preference.

The above applies to an individual person; hence the $p$ subscript is used. However, a substantial literature is based on the concept of a ‘representative individual’. In this case, optimal plans are unambiguously based on the preferences of this representative individual and there are no difficulties over the aggregation of preferences.\textsuperscript{12} In fact, macroeconomic models using a so-called representative individual are strictly speaking constructed in terms of just one individual: there is no consideration of aggregation requirements.\textsuperscript{13}

**Social Evaluations**

In practical social evaluations or cost benefit evaluations involving multi-period contexts, it is often inappropriate to think of a single individual. Evaluations may be based on a social welfare function, as in the single-period case. A choice of welfare metric for each period must be made. In the following discussion this is taken to be aggregate consumption, which in the case of a fixed population size translates simply to consumption per person.\textsuperscript{14}

Hence, where the $C$s now represent aggregates, the welfare function can be written as:

$$W_T = \sum_{t=1}^{T} \left( \frac{1}{1 + \rho_m} \right)^{t-1} W(C_t)$$  \hfill (13)

\textsuperscript{12} In some growth models, the representative individual is assumed to be infinitely lived. In order to prevent the individual simply accumulating huge debts, a condition is imposed such that the rate of interest and the individual’s discount rate are equal. In turn, this means that $\varepsilon p g$ and $\rho$ cannot be set independently, given $g$.

\textsuperscript{13} From the demand analysis literature, it is known that quasi-homothetic (Gorman) preferences are required for aggregates to be interpreted as arising from the preferences of a representative individual. In this case demands are linear, with a common slope, although intercepts may differ to allow for, say, demographic factors.

\textsuperscript{14} The questions of the welfare metric and the unit of analysis in the multi-period case are examined in detail by Creedy and Guest (2006).
In this case, \( W(C_i) \) represents the contribution of period \( i \)’s aggregate consumption to the evaluation function, and \( \rho_m \) represents the pure time preference rate of the hypothetical judge. The corresponding ubiquitous iso-elastic function is thus:

\[
W(C_i) = \frac{C_t^{1-\rho_m}}{1-\epsilon_m}
\]  

(14)

In this context the discount rate is commonly referred to as the ‘social time preference rate’, \( r_m \), and is given, where \( g_m \) is the aggregate growth rate of consumption, by:

\[
r_m = \rho_m + \epsilon_m g_m
\]  

(15)

Although equations (12) and (15) have the same basic form, there are no grounds for taking values appropriate to one and substituting in the other. Again, it must be stressed that the term \( \epsilon_m \) represents the value judgements of a judge or policy maker. It may be possible to obtain empirical estimates of \( \epsilon_p \) using data on saving behaviour over time for a sample of individuals; on the approach used, with a review of alternative estimates, see Pearce and Ulph (1998). However, there are no grounds whatsoever for imposing \( \epsilon_m = \epsilon_p \); the former involves a value judgement and there is no logical connection between the two rates.\(^{15}\)

In thinking about the appropriate values for \( \epsilon_m \) in this context, quite different considerations apply compared with the case of single-period distributional judgements involving inequality aversion, \( \epsilon_s \), discussed earlier. The term \( \epsilon_m \) is, in the multi-period framework, more accurately interpreted in terms of an aversion on the part of the judge towards variability - inequality between periods rather than inequality between persons. Yet unfortunately these terms are often conflated in the literature, where discussion proceeds as if \( \epsilon_m, \epsilon_p, \epsilon_{LEP} \) and \( \epsilon_s \) were measuring the same thing.

Although the emphasis of this paper is on the elasticity of marginal valuation, a further warning is worth sounding in the multi-period context. This is because an additional value judgement is needed for evaluations of the welfare function: this concerns the pure time preference rate of the hypothetical judge, \( r_m \). Instead of examining the implications of adopting alternative value judgements by looking at a range of values – the legitimate role of the professional economist – it is not uncommon to find authors using standard rhetorical devices to impose their own value judgements. Thus dogmatic statements along the lines that ‘positive pure time preference is morally indefensible’ can often be found. However, it is worth remembering that zero pure time preference carries the implication that the judge would be prepared to impose starvation on the current generation in order to produce a tiny benefit for a distant generation.

4. Taxation and Equal Absolute Sacrifice

The previous sections of this paper have criticised attempts to use empirical estimates, based on samples of households’ observed consumption behaviour, as the basis of an

\(^{15}\)Marina and Scaramozzino (2000, p.6) provided an interesting analysis of growth in an overlapping generations framework. They stated that, ‘a social rate of pure time preference is justifiable on purely ethical grounds’. A clearer statement of what the authors showed is that if the objective of maximising average steady-state consumption per capita is adopted, then an implication of this ethical value judgement, combined with a model containing productivity and population growth, is that positive time preference exists that does not reflect myopia.
argument over values which ‘should’ be imposed in social evaluations. The present section turns to a quite different, but also illegitimate, approach involving attempts to estimate the implicit value judgements revealed by tax and transfer policies. Some authors, such as Stern (1977), Cowell and Gardiner (1999) and Evans (2005), have suggested that estimates provide a guide to $\varepsilon$ values which should be applied in policy evaluations.

There are two steps to such an approach. The first step attempts to infer, from tax policy decisions, value judgements which are not otherwise made explicit. Such an attempt can be defended – provided of course that the model used to make inferences is plausible. For example, such estimates may be useful in checking whether there is in fact any correspondence between policies and basic value judgements of policy makers. Given the complexities involved in tax policy design, it may be useful to know if a particular structure is associated with implicit judgements that may actually be very different from those held (though seldom made explicit). This is the view taken by van de Ven and Creedy (2005) when examining adult equivalence scales implicit in tax and transfer systems.

The second step is the wholly illegitimate one of suggesting that estimates of implicit value judgements ‘should’ be used in making social evaluations. This criticism applies even in the most unlikely case where implicit views can be identified precisely.

This section considers the first ‘positive’ step taken by the authors mentioned above, and suggests that the model used is inadequate. The approach is based on the assumption that income tax policy-makers aim to achieve equal absolute sacrifice. It assumes that incomes are exogenously given, rather than arising from endogenous labour supply behaviour (subject to endowments and education which give rise to individual productivities). Suppose $x$ represents income and the tax function is $T(x)$. Equal absolute sacrifice requires, for all $x$, that the absolute difference between pre-tax and post-tax utility is the same for all individuals. Hence:

$$U(x) - U(x - T(x)) = k$$

where $U(.)$ represents a utility function which is considered to be the same for all individuals. The parameter $k$ depends on the amount of revenue per person.\(^\text{16}\) The combination of equal absolute sacrifice with the iso-elastic function, $U(x) = x^{1-\varepsilon_t}/(1 - \varepsilon_t)$ for $\varepsilon_t \neq 1$ gives, from (16) above:\(^\text{17}\)

$$\frac{x^{1-\varepsilon_t}}{1 - \varepsilon_t} - \frac{(x - T(x))^{1-\varepsilon_t}}{1 - \varepsilon_t} = k$$

Differentiation and simplification gives, as in Evans (2005, p.207), the result that:

$$\log (1 - T'(x)) = \varepsilon_t \log \left(1 - \frac{T(x)}{x}\right)$$

\(^\text{16}\) This differs from an alternative view that would replace $U(x)$ with $W(x)$. Thus, as with inequality measurement, a judgement is made regarding the welfare metric, and then a view is taken about variations in $x$. This judgement is quite separate from the way individuals may themselves view such variations.

\(^\text{17}\) Young (1987) actually showed that the iso-elastic form is required if an indexation requirement is imposed on the tax structure in addition to equal sacrifice. But of course fiscal drag is a common, indeed almost universal, feature of income tax structures.
where $T'(x)$ and $T(x)/x$ are marginal and average tax rates. This expression has been used to carry out ordinary least squares regressions using income tax schedules, so that $\varepsilon_t$ and its standard error are obtained as a regression coefficient. There is some difference of opinion over whether to include a constant in the regression: compare Cowell and Gardiner (1999) and Evans (2005), who also use different income measures. Alternatively, (18) can be rearranged to get $\varepsilon_t = \log (1 - MTR)/\log (1 - ATR)$ and ‘estimates’ of $\varepsilon_t$ are obtained and compared using simply the marginal and average tax rates at different income levels.

The first point to stress regarding this approach is that it automatically produces a value of $\varepsilon_t$ in excess of unity for a progressive tax system, for which the marginal tax rate exceeds the average tax rate. This feature was first discussed by Edgeworth (1897) and formally shown by Samuelson (1947). The values of $\varepsilon_t$ obtained in this way are thus severely constrained by the specification of the objective of equal absolute sacrifice. Furthermore, those using the approach to ‘estimate’ $\varepsilon$ ignore the objections raised by Edgeworth and others concerning the various interpretations of sacrifice theories. For $\varepsilon_t >$ equation (17) can be rearranged as:

$$T(x) = x - \{x^{1-\varepsilon_t} - k (1 - \varepsilon_t)\}^{1/(1-\varepsilon_t)}$$

(19)

Examples of marginal and average rate schedules based on this function are shown in figure 2. Of course, in practice tax functions are multi-step functions with ranges where the marginal rate is constant. In structures like that in the UK, there is a ‘standard rate’ which applies over a wide range of taxable income, so the above function obviously has difficulty capturing this range. The imposition of $e >$ is also highly restrictive (as with the use of the linear expenditure system mentioned above). Furthermore, the model applies only to positive taxes. It can thus relate at best to a small component of a much broader set of taxes and transfers.

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18 This does not apply to those, such as Richter (1983) and Young (1987) who were interested only in deriving the implications of various axioms.

19 In stating this result, Young (1987, p. 212) rewrote $-k (1 - \varepsilon)$ as $\lambda^{1-\varepsilon}$, so that the tax function compares with a constant elasticity of substitution form.

20 The coefficient $k$ is determined by the amount of revenue raised by the tax. Suppose that $x$ follows a lognormal distribution with mean and variance of logarithms of $\mu = 10$ and $\sigma^2 = 0.5$ respectively. These values imply an arithmetic mean income of $52,828. Suppose it is required to raise revenue per person of $10,000 and that $\varepsilon_t = 1.5$. Using a numerical iterative search procedure, it is found that this requires $k = 0.0025$. This is based on a simulated population obtained from 5000 random draws from the assumed income distribution.

21 If equal absolute sacrifice is combined with a welfare function displaying constant absolute inequality aversion, $\alpha$, such that $W = 1 - \exp (-\alpha x)$ a tax function of the form $T(x) = x + \frac{1}{\alpha} \log \{k + e^{-\alpha x}\}$ arise. This can be made to display rate schedules similar to those illustrated above. Dalton (1954, pp.68-70) discussed several examples using alternative utility functions and sacrifice principles, and showed that if equal absolute sacrifice produces progression, equal proportional sacrifice produces a more progressive tax structure. In an early study, Preinreich (1948) considered the form of the utility schedule consistent with the US tax legislation, without imposing a specific functional form over the whole income range. He assumed equal proportional sacrifice.
Hence some scepticism must be attached to interpretations of estimates obtained using this model as implicit value judgements. It seems most likely that the approach has been chosen largely — or indeed only — for its simplicity. On the other hand, the optimal tax framework has demonstrated the considerable complexity involved in the link between value judgements and the tax structure and, importantly, progression can arise with values of $\varepsilon < 1$. But of course even if the estimation of implicit preferences were considered plausible, they cannot qualify as value judgements which should be imposed. There is no alternative to accepting that value judgements are required and the best attitude of professional economists is to report a range of results based on alternative value judgements. In reporting results, readers need to appreciate precisely what is implied about value judgements by different values of $\varepsilon$, since it is not immediately obvious whether, for example, a value of $\varepsilon = 0.5$ indicates a high or low aversion to inequality. This is considered in the following section.

5. Interpreting Orders of Magnitude

In examining the implications of alternative value judgements, using an iso-elastic weighting function with different values of $\varepsilon$, it is important to appreciate the precise nature of the comparisons being made. When the link between this type of social welfare function and a measure of inequality was introduced by Atkinson (1970), he recognised the difficulty of forming views about the orders of magnitude of $\varepsilon$ using the welfare function $W = \sum_{h=1}^{H} \frac{y_{h}^{1-\varepsilon}}{1-\varepsilon}$. In order to help interpretation, he used the idea of a ‘leaky bucket’ experiment, which considers the extent to which a judge is prepared to tolerate some loss in making a transfer from one person to another.\(^{22}\)

Consider two individuals, so that from the welfare function, setting the total differential equal to zero gives:

\(^{22}\) Okun (1975) examined a slightly different kind of leaky bucket experiment involving transfers between groups of individuals.
The welfare function is thus homothetic, as the slopes of social indifference curves are the same along any ray drawn through the origin. Consider two individuals and, using discrete changes, suppose a dollar is taken from the richest, such that \( \Delta y_2 = -1 \). The amount to be given to the other individual to keep social welfare unchanged is thus:

\[
\Delta y_1 = \left( \frac{y_1}{y_2} \right)^\varepsilon
\]

For example, if \( y_2 = 2y_1 \) and \( \varepsilon = 1.5 \), it is necessary to give person 1 only 35 cents – a leak of 65 cents from the original dollar taken from person 2 is tolerated. If \( \varepsilon = 1 \), a leak of 50 cents is tolerated.

This type of experiment, and thus the sensitivity of the tolerance for a leaking bucket, is well-known in the literature on inequality measurement. But in other contexts in which the same kind of iso-elastic function is used, relatively large values of \( \varepsilon \) are often adopted without, it seems, consideration of such implications.\(^{23}\) For example, in the intertemporal literature, a value of \( \varepsilon = 2 \) is often used. Suppose that total income (or consumption) in the first period is 100 and this grows at a rate of 0.02 per period. In period 10 it is thus 119.5, and a judge with \( \varepsilon = 2 \) would be prepared to take a dollar from period 10, and give only $0.70 to period 1. By period 20 total income would be 145.7, and the same judge would reduce period 20’s income by $1 while adding only $0.47 to the first period. The social time preference rate is thereby increased significantly above the pure time preference rate.

The leaky bucket experiment therefore provides a useful illustration of the implications, in terms of value judgements, of adopting particular values of \( \varepsilon \) in any policy evaluation.

### 6. Conclusions

This paper has been concerned with the use of social welfare functions in evaluating actual or potential changes resulting from policies or other factors affecting a well-defined group of individuals. In particular, it has considered suggestions regarding the welfare weights to be used in comparing the gains and losses of different individuals (or other appropriate units of analysis), and a social time preference rate for use in cost benefit evaluation. While these variables essentially reflect value judgements, some authors have argued that they can be estimated either from consumers’ behaviour or from the judgements implicit in tax policy. It was instead suggested here that results are highly sensitive to the context and model specification assumed.

More importantly, the argument that an estimated elasticity of marginal utility or social time preference rate should be used in policy evaluations fails to recognise that fundamental value judgements are involved. The various estimates and models

\(^{23}\) However, it is discussed by Pearce and Ulph (1998, pp.280-281).
may be of interest, but they cannot be used by economists to impose value judgements. The main contribution economists can make is to examine the implications of adopting a range of alternative value judgements.

This argument is of course not new. Indeed it was stated most forcefully and eloquently by Robbins (1935) in his important book on the Nature and Significance of Economic Science. He argued, ‘propositions involving the verb ‘aught’ are different in kind from propositions involving the verb ‘is’. And it is difficult to see what possible good can be served by not keeping them separate, or failing to recognise their essential difference’ (1935, p.149). It seems worthwhile to repeat these warnings of Robbins, along with his view that:

All this is not to say that economist may not assume as postulates different judgments of value, and then on the assumption that these are valid enquire what judgment is to be passed upon particular proposals for action. On the contrary, as we shall see, it is just in the light that it casts upon the significance and consistency of different ultimate valuations that the utility of economics consists (1935, p.149).

References


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